

# A Model of Delegated Project Choice\*

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## Abstract

We present a model in which a principal delegates the choice of project to an agent with different preferences. The principal determines the set of projects from which the agent may choose. The principal can verify the characteristics of the project chosen by the agent, but does not know which other projects were available to the agent. We consider situations where the collection of available projects is exogenous to the agent but uncertain, where the agent must invest effort to discover a project, where the principal can pay the agent to choose a desirable project, and where the principal can adopt more complex schemes than simple permission sets.

**Keywords:** Delegation, principal-agent, rules, merger policy.

## 1 Introduction

In the main model in this paper we present an analysis of a principal-agent problem in which the principal can influence the agent's behaviour not by outcome-contingent rewards but by specifying what the agent is, and is not, allowed to do. The agent, whose preferences differ from those of the principal, will select from her available projects the permitted project that best serves her interests. The principal can verify whether or not the selected project is indeed within the permitted set, but cannot observe the number or characteristics of the other projects available to the agent.

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This paper investigates how the principal should best specify the set of projects from which the agent can choose.

An application of our analysis is to an important issue in competition policy, which is the appropriate welfare standard to use when evaluating mergers (or some other form of conduct). The two leading contenders are a *total welfare* standard, where mergers are evaluated according to whether they decrease the unweighted sum of producer and consumer surplus, and a *consumer welfare* standard, where mergers detrimental to consumers are blocked. Many economic commentators feel that antitrust policy should aim to maximize total welfare, whereas in many jurisdictions the focus is more on consumer welfare alone. See Farrell and Katz (2006) for an overview of the issues. One purpose of this paper is to examine a particular strategic reason, discussed previously by Lyons (2002) and Fridolfsson (2007), to depart from the regulator's true welfare standard, which is that a firm may have a *choice* of merger possibilities. A less profitable merger might be better for total welfare, but will not be chosen under a total welfare standard. To illustrate, consider Figure 1.

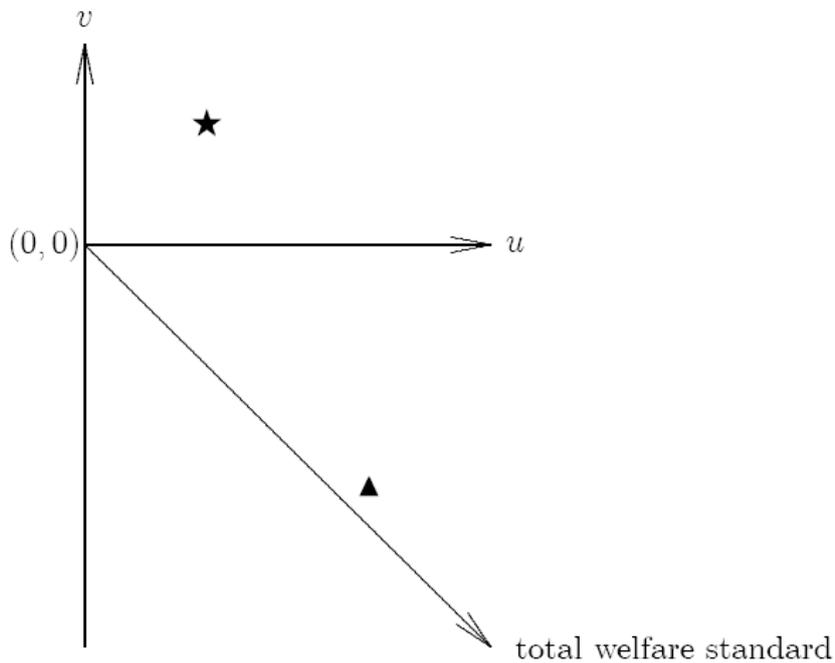


Figure 1: The Impact of Welfare Standard on Chosen Mergers

Here,  $u$  represents the gain in total profit resulting from a particular merger, while  $v$  measures the resulting gain (which may be negative) to consumers. Suppose that

$u$  and  $v$  are verifiable once a merger is proposed to the competition authority. If the regulator follows a total welfare standard, he will permit any merger which lies above the negatively-sloped line in the figure. Suppose the firm can choose between the two mergers depicted by  $\blacktriangle$  and  $\blackstar$  on the figure. With a total welfare standard, the firm will choose the merger with the higher  $u$  payoff, i.e., the  $\blacktriangle$  merger. However, the regulator would prefer the alternative  $\blackstar$  since that yields higher total welfare. If the regulator instead imposed a consumer welfare standard, so that only those mergers which lie above the horizontal line  $v = 0$  are permitted, then the firm will be forced to choose the preferred merger. In this case, a regulator wishing to maximize total welfare is better off if he imposes a consumer welfare standard. As Farrell and Katz (2006, page 17) put it: “if we want to maximize gains in total surplus (northeasterly movements as shown in Figure [1]) and firms always push eastwards, there is something to be said for someone adding a northerly force.” Nevertheless, there is a potential cost to adopting a consumer welfare standard: if the  $\blacktriangle$  merger turns out to be the *only* merger possibility then a consumer welfare standard will not permit this even though the merger will improve total welfare. Thus, the choice of welfare standard will depend on the likely number of possible mergers and the distribution of profit and consumer surplus gains for a possible merger.<sup>1</sup>

Another application of our analysis could be to project choice within an organization. While shareholders wish to choose the available project which yields the highest net present value (NPV), a manager might prefer larger, more capital intensive projects. If the manager sometimes has a choice of project and has the ability to hide her less preferred projects, shareholders may wish to limit the kinds of projects which can be implemented. This question is analyzed by Berkovitch and Israel (2004), who write [page 241]: “[I]f headquarters cannot observe all available projects, then the manager may manipulate the selection process by presenting projects such that managerial utility is maximized. [...] [W]hile NPV is the best way to *measure* value added, in many situations, it is not a good way to *implement* the selection of the

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<sup>1</sup>Besides administrative filing fees, merging firms do not make monetary payments to competition authorities, still less payments contingent on merger approval. Approval is sometimes made conditional upon the implementation of (non-monetary) remedies, such as the sale to third parties of some assets. Our schematic framework would need adaptation to allow for this, although the issue of the appropriate welfare standard nevertheless applies to the questions about merger remedies in that they are designed to ensure that mergers are not likely to be detrimental in terms of the chosen standard.

highest NPV projects.”

More generally, our analysis addresses an aspect of the theory of optimal *rules*: the relationship between the ultimate objective of the rule-setter and the optimal rule to commit to. That relationship is not straightforward inasmuch as the likely consequences of a rule, including for the attainment of the ultimate objective, depend on the responses of agents seeking to maximize, within the rules, their own objectives. The interplay between rules and the responses that they induce is at the heart of our analysis. Our goal is to characterize optimal rules—which are sometimes strikingly simple—in terms of the fundamentals of our models.

Our benchmark model in section 3 analyzes a setting in which the agent chooses one project from an exogenous, but uncertain, finite set of available projects. Monetary incentives are ruled out, and the principal optimally restricts agent choice in a way that forbids some projects that are moderately good, in the hope of inducing the agent instead to choose a project that is better for the principal. This bias is akin to putting less weight on the agent’s payoff than is in the true welfare function.

In section 3 we present three variants of this benchmark model. In section 3.1 we analyze a setting where the agent influences the likelihood of finding a project by exerting costly effort. Here, we show that the principal optimally sets a *linear* permission rule. In addition, to induce greater effort, the principal allows some projects that are detrimental for his interests; this bias is akin to putting more weight on the agent’s payoff than is in his true welfare function. Second, in section 3.2 we discuss the impact of monetary incentives to choose a good project. When the agent is liquidity constrained, it may be preferable to restrict the agent’s freedom to choose projects than to reward her for choosing a good project. Finally, in section 3.3 we consider the possible benefits to the principal of using a more complex delegation scheme. For instance, the principal may sometimes do better if he permits a mediocre project to be implemented when the agent reports she has several other mediocre projects available. However, when the number of projects comes from a Poisson distribution, the principal can do no better than to offer a simple permission set.

Some other papers have examined situations in which a principal delegates decision-making to a (potentially) better-informed agent whose preferences differ from those of the principal, and where contingent transfers between principal and agent are ruled

out. Aghion and Tirole (1997) show how, depending on information structure and payoff alignment, it may be optimal for a principal to delegate full decision-making power to a potentially better-informed agent. The principal’s loss of control over project choice can be outweighed by advantages in terms of encouraging the agent’s initiative to develop and gather information about projects. In like vein Baker, Gibbons, and Murphy (1999), though they deny formal delegation of authority, examine informal delegation through repeated-game relational contracts. Even an informed principal able to observe project payoffs may refrain from vetoing ones that yield him poor payoffs in order to promote search incentives for the agent.

Our work is closer to the models which analyze *constrained* delegation, where the agent can make decisions but only within specified limits and the principal’s problem is to decide how much leeway to give the agent. (For instance, a judge sets a convicted criminal’s punishment, but only within mandatory minimum and/or maximum limits for the type of crime.) This literature was initiated by Holmstrom (1984), and the elements of his model go as follows. There is a set of decisions, indexed by a scalar variable  $d$  which takes values in some interval  $D$ , one of which needs to be made. Unlike our model where the agent must choose from a finite set of projects, here *any* decision is feasible. A given decision generates payoffs to the two parties which depend on the state of the world, represented by  $\theta$ , and only the agent observes this parameter. The preferences of the principal and agent differ, and if decision  $d$  is made when the state is  $\theta$  the principal obtains payoff  $V(d, \theta)$  and the agent has payoff  $U(d, \theta)$ . The principal’s problem is to choose a set, say  $\mathcal{D} \subset D$ , from which the agent is permitted to choose her decision. This permission set is chosen to maximize the principal’s expected payoff (given his prior on  $\theta$ ), given that the agent will make her preferred decision from  $\mathcal{D}$  given the state  $\theta$ .<sup>2</sup>

Holmstrom mostly limits attention to cases where the permission set  $\mathcal{D}$  is an interval. Subject to this assumption (and other regularity conditions), he shows that an agent whose preferences are closer to the principal’s will be given wider discretion. (This result has subsequently sometimes been termed the “ally principle”.)

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<sup>2</sup>This “delegation problem” coincides with the “mechanism design problem” where the agent makes an announcement about the claimed state of the world,  $\hat{\theta}$ , and the principal commits to a rule  $d(\hat{\theta})$  which maps the announcement to the implemented decision. The two approaches are equivalent since the principal never directly observes the true  $\theta$  and by making a suitable announcement  $\hat{\theta}$  the agent can implement any decision in the range of the rule  $d(\cdot)$ .

Following Holmstrom’s initial contribution, subsequent papers have analyzed when interval delegation is optimal for the principal, making the additional assumption that  $\theta$  is a scalar variable.<sup>3</sup> Melumad and Shibano (1991) were the first to calculate optimal permission sets, in the special case where preferences were quadratic and where  $\theta$  was uniformly distributed. They found that interval delegation was optimal when principal and agent have ideal policies which are similarly responsive to the state  $\theta$ , but that otherwise it could be optimal to have “holes” in  $\mathcal{D}$ . Martimort and Semenov (2006) find a sufficient condition on the distribution of  $\theta$  for interval delegation to be optimal. Alonso and Matouschek (2008) systematically investigate when interval delegation is optimal, and they generalize Melumad and Shibano’s insight that the relative responsiveness of preferred decisions to the state is the key factor for this. They show that when interval delegation is sub-optimal the ally principle need not hold and an agent with preferences more aligned with those of the principal might optimally be given less discretion.

Those models in the Holmstrom tradition differ from ours in respect of the actions which are feasible and the form of asymmetric information. In particular, they characterize each decision by a scalar parameter (such as the length of a prison sentence), all decisions are always feasible, and the agent has private information about a payoff-relevant state of the world. In our model, by contrast, payoffs of the chosen project to both principal and agent are known, but only a finite number of projects are feasible (such as the possible mergers for a firm) and only the agent knows what those possible projects are. Like the papers discussed above, our aim is to characterize the optimal permission set from which the agent can choose, but in a two-dimensional setting where the principal can observe both his own and the agent’s payoff from the project chosen by the agent.<sup>4</sup>

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<sup>3</sup>Szalay (2005) presents an interesting variant on this delegation problem in which interval delegation is often sub-optimal. In his model, there is no divergence in preferences between the principal and agent, but the agent incurs a private cost to observe  $\theta$ . He shows that it can be optimal for the principal to remove intermediate policies from  $\mathcal{D}$  so that the agent is forced to choose between relatively extreme options, as this sharpens the agent’s incentive to discover  $\theta$ .

<sup>4</sup>A paper which also investigates a two-dimensional delegation problem is Amador, Werning, and Angeletos (2006). There, an agent with quasi-hyperbolic preferences has wealth which she consumes over two periods. If there were no uncertainty about her preferences, she would gain by committing to a fixed consumption path at time zero. However, she will receive a utility shock in period 1 and this uncertainty gives a motive to allow some flexibility in consumption. Amador *et al.* find a condition which implies that the optimal delegation set simply involves placing a ceiling on first-period consumption.

## 2 Benchmark Model: Choosing a Project

A principal (“he”) delegates the choice of project to an agent (“she”). There may be several projects for the agent to choose from, although only one can be implemented over the relevant time horizon. A project is fully described by two parameters,  $u$  and  $v$ . The agent’s payoff if the type- $(u, v)$  project is implemented is  $u$ , while the payoff to the principal, who is assumed to be risk-neutral, is  $v + \alpha u$ . (If no project is implemented, each party obtains payoff of zero.) Here,  $\alpha \geq 0$  represents the weight the principal places on the agent’s interests, and  $v$  represents factors specific to the principal’s interests. The parameter  $\alpha$  might reflect a true regard for the agent’s payoff (as in the merger application when profits carry some weight in social welfare), and/or it might reflect a trade-off between allowing the agent wider project choice—and so a greater chance of on-the-job benefits  $u$ —and paying her a higher (non-contingent) salary.<sup>5</sup> For example, if the principal jointly chooses the permitted set of projects and the salary to meet the participation constraint of a risk-neutral agent, and  $u$  is measured in money terms, then the following analysis applies with  $\alpha = 1$ .<sup>6</sup>

Each project is an independent draw from the same distribution for  $(u, v)$ . Since without contingent money rewards the agent will never propose a project with a negative payoff, without loss of generality we suppose that only non-negative  $u$  are realized. The marginal density of  $u \geq 0$  is  $f(u)$ . The conditional density of  $v$  given  $u$  is denoted  $g(v, u)$  and the associated conditional distribution function for  $v$  is  $G(v, u)$ . Here,  $v$  can be positive or negative. Suppose that the support of  $(u, v)$  is a rectangle  $[0, u_{\max}] \times [v_{\min}, v_{\max}]$ , where  $v_{\min} \leq 0 \leq v_{\max}$  so that  $(0, 0)$  lies in the support of  $(u, v)$ . Finally, suppose that both  $f$  and  $g$  are continuously differentiable and non-zero on the support of  $(u, v)$ .

In this benchmark model, the number of projects is random and the probability that the agent has exactly  $n \geq 0$  available projects is  $q_n$ . (Our analysis covers the case where there are surely  $N$  projects, but the analysis is no easier for that case.

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<sup>5</sup>Aghion and Tirole (1997) refer to this benefit of giving the agent freedom to choose projects as the “participation” benefit of delegation.

<sup>6</sup>At the other extreme, if the agent is infinitely risk-averse and cares only about her minimum income over all outcomes, then the following analysis applies when  $\alpha = 0$ . Such an agent is unwilling to trade-off her salary against the uncertain prospect of on-the-job benefits.

Indeed, we will see that  $n$  being a Poisson variable is the easiest example to analyze.) Suppose that the project characteristics  $(u, v)$  are distributed independently of  $n$ .

The principal delegates the choice of project to the agent. We assume in this benchmark model that the principal cannot offer contingent monetary incentives to the agent to choose a desirable project, and also that the characteristics of the chosen project—and *only* that project—are verifiable. This latter assumption may be appropriate if the principal can verify the claimed project characteristics only after the project has been implemented. (Suppose that the principal has the ability to punish the agent if it turns out that the agent misrepresented the payoffs.) Alternatively, the assumption is reasonable if the principal has substantial costs associated with auditing each project’s characteristics, and/or the agent has significant costs associated with preparing a credible proposal for an additional project. (Each of these is arguably the case in the merger scenario, for example.) We also assume that the principal considers only deterministic policies.<sup>7</sup>

Under the assumption that only the chosen project’s characteristics are verifiable, the principal’s (deterministic) problem reduces to the delegation problem of choosing a set of permitted projects.<sup>8</sup> That is, before the agent has any private information, the principal commits to a (measurable) permission set of projects, denoted  $\mathcal{D} \subset [0, u_{\max}] \times [v_{\min}, v_{\max}]$ , and the agent can then implement any project that lies in  $\mathcal{D}$ .<sup>9</sup> In section 3.3 we discuss an alternative, and sometimes superior, delegation

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<sup>7</sup>We restrict attention to deterministic policies mainly because it is hard to imagine being able to commit to or implement a stochastic mechanism in practice. It is possible that a stochastic scheme, if feasible, could do better than a deterministic scheme. For example, suppose that  $\alpha = 0$ , that  $n = 2$  for sure, and that  $(u, v) = (0.5, 1)$  with probability 0.5 and  $(u, v) = (0.9, 0.1)$  with the same probability. In this case, the optimal deterministic scheme only permits the principal’s favoured project,  $(0.5, 1)$ . However, permitting the project  $(0.9, 0.1)$  with probability 0.5 yields the principal a higher expected payoff than banning it altogether: in both cases the agent would choose the principal’s preferred project if she could, but if that is not available the stochastic scheme would still allow some chance of a desirable project being chosen.

<sup>8</sup>Under this assumption, similarly to footnote 2, this delegation approach is equivalent to a mechanism design approach in which the principal commits to a rule that determines which project is chosen as a function of the agent’s report of her private information, i.e., the number and characteristics of available projects. To see this, note that the set of projects can be partitioned into two subsets: the set of projects, say  $\mathcal{D}$ , which, by making suitable reports of other projects, could be chosen for implementation under the principal’s decision rule, and those projects which are never implemented by the principal’s rule. Faced with this rule, the agent will simply choose her preferred available project (if any) in the former set, and announce any other projects required to implement that choice. Clearly, this mechanism is equivalent to the delegation problem where the agent can directly choose any project in the set  $\mathcal{D}$ .

<sup>9</sup>It is important to emphasize the assumption here, as in delegation problems more generally,

scheme which can be used when the principal can cheaply verify a *list* of projects which the agent reveals to be available.

Given a permission set  $\mathcal{D}$ , for each  $u$  let  $\mathcal{D}_u = \{v \text{ such that } (u, v) \in \mathcal{D}\}$  be the set of type- $u$  projects which are permitted, and let

$$p(u) = \int_{v \in \mathcal{D}_u} g(v, u) dv$$

be the proportion of type- $u$  projects which are permitted. Let

$$x(u) = 1 - \int_u^{u_{\max}} p(z) f(z) dz$$

to be the probability that any given project either has agent payoff less than  $u$  or is not permitted. Note that  $x(0)$  is the fraction of project types which are banned, that  $x(\cdot)$  is continuous, and when differentiable its derivative is

$$x'(u) = p(u) f(u) . \quad (1)$$

(Since  $x(\cdot)$  is weakly increasing it is differentiable almost everywhere.) If there are  $n$  available projects, the probability that each project is either banned or generates agent payoff less than  $u$  is  $(x(u))^n$ . Summing over  $n$  implies that the probability that each available project is either banned or generates agent payoff less than  $u$  is  $\phi(x(u))$ , where

$$\phi(x) \equiv \sum_{n=0}^{\infty} q_n x^n$$

is the *probability generating function* (PGF) associated with the random variable  $n$ . It follows that the density of the agent's preferred permitted project (where this exists) is  $\frac{d}{du} \phi(x(u))$ . Useful properties of PGFs are that they are well-defined on the relevant interval  $0 \leq x \leq 1$  and smooth, convex and increasing over this interval.

The principal's payoff with permission set  $\mathcal{D}$  is therefore

$$\begin{aligned} & \int_0^{u_{\max}} \{E[v \mid (u, v) \in \mathcal{D}] + \alpha u\} \frac{d}{du} \phi(x(u)) du \\ &= \int_0^{u_{\max}} \left\{ \int_{v \in \mathcal{D}_u} v g(v, u) dv + \alpha u p(u) \right\} f(u) \phi'(x(u)) du . \end{aligned} \quad (2)$$

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of commitment. (However, see section 3.3 below for discussion about how the principal does not want to renegotiate the permission set in the case where the number of projects follows a Poisson distribution.) Baker, Gibbons, and Murphy (1999) and Alonso and Matouschek (2007) show how the principal's commitment power can be endogenously generated with repeated interaction (as is the case in the merger context, for instance).

The principal's problem is to maximize expression (2) taking into account the relationship between  $p$  and  $x$  in (1) and the endpoint constraint  $x(u_{\max}) = 1$ . The following lemma shows that the optimal permission set takes a “threshold” form:

**Lemma 1** *In the optimal policy there exists a threshold rule  $r(\cdot)$  such that*

$$(u, v) \in \mathcal{D} \text{ if and only if } v \geq r(u) .$$

**Proof.** From (1), the function  $x(\cdot)$  depends on  $\mathcal{D}$  only via the “sufficient statistic”  $p(u)$ , not on the particular  $v$ -projects which are permitted given  $u$ . Therefore, for any candidate function  $p(u)$  the principal might as well permit those particular  $v$ -projects which maximize the term  $\{\cdot\}$  in the (2), subject to the constraint that the proportion of type- $u$  projects is  $p(u)$ . But the problem of choosing the set  $\mathcal{D}_u$  in order to

$$\text{maximize } \int_{v \in \mathcal{D}_u} vg(v, u)dv \text{ subject to } \int_{v \in \mathcal{D}_u} g(v, u)dv = p(u)$$

is solved by permitting the projects with the highest  $v$  so that the proportion of permitted projects is  $p(u)$ , i.e., that  $\mathcal{D}_u = \{v \text{ such that } v \geq r(u)\}$  for some  $r(u)$ . ■

Thus, the problem simplifies to the choice of threshold rule  $r(\cdot)$  rather than the choice of permission set  $\mathcal{D}$ . (A similar argument is valid in the project discovery model in section 3.1.) Figure 2 depicts a threshold rule  $r(\cdot)$ , and shows  $x(u)$  as the measure of the shaded area.

Define  $V(r, u)$  to be the expected value of  $v$  given that the project has agent payoff  $u$  and that  $v$  is at least  $r$ . Recasting (2) in terms of  $r(\cdot)$  rather than  $\mathcal{D}$ , the principal's problem is to choose  $r(\cdot)$  to maximize

$$\int_0^{u_{\max}} [V(r(u), u) + \alpha u][1 - G(r(u), u)]f(u)\phi'(x(u)) du \quad (3)$$

subject to the “equation of motion”

$$x'(u) = [1 - G(r(u), u)]f(u) \quad (4)$$

and the endpoint condition

$$x(u_{\max}) = 1 . \quad (5)$$

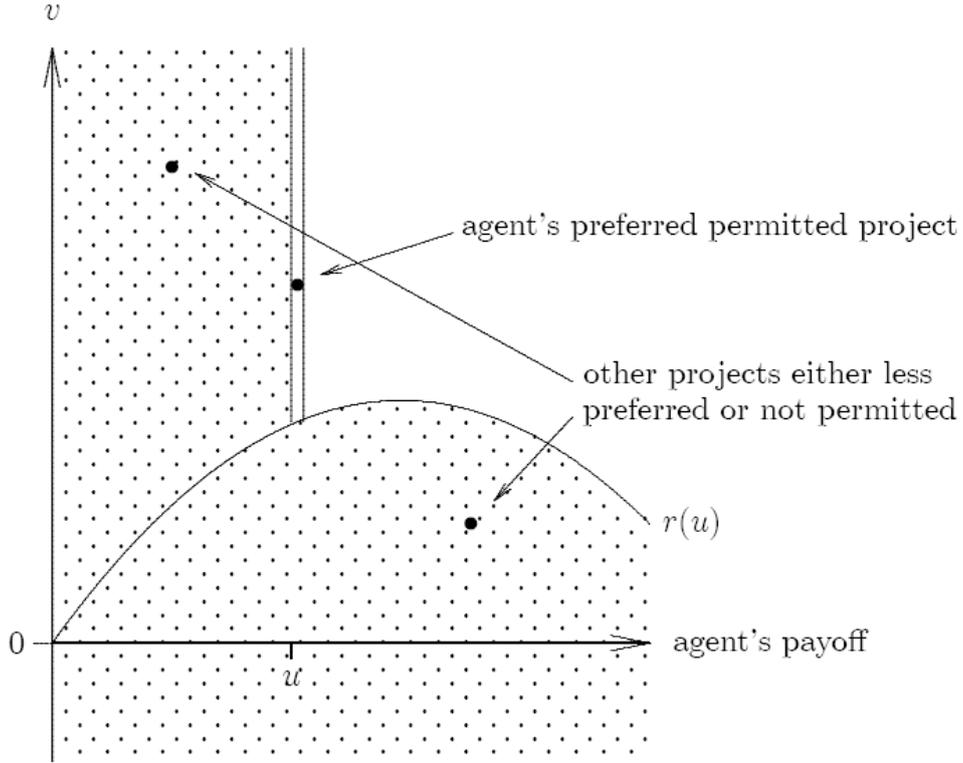


Figure 2: The Agent's Preferred Permitted Project

This optimal control problem is solved formally in the appendix, but its solution can be understood intuitively with the following argument. Consider a point  $(u, r(u))$  on the frontier of the permitted set. For the set to be optimal it is necessary that the principal be indifferent between his payoff  $[r(u) + \alpha u]$  at that point and his expected payoff from the agent's next-best permitted alternative, conditional on the agent's best permitted project being  $(u, r(u))$ .

To calculate the latter expected payoff, note that for a given project the density of payoff vector  $(u, r(u))$  is  $f(u)g(r(u), u)$ . Then the probability that one out of  $n$  projects has payoffs  $(u, r(u))$  and all the others have agent utility no greater than  $z \leq u$  or are not permitted is  $n f(u)g(r(u), u)[x(z)]^{n-1}$ . Taking the sum across  $n$ , the probability that one project has payoffs  $(u, r(u))$  and all other permitted projects have utility no greater than  $z$  is therefore  $f(u)g(r(u), u)\phi'(x(z))$ . In particular, the probability that the agent's preferred permitted project is  $(u, r(u))$  is  $f(u)g(r(u), u)\phi'(x(u))$ . Conditional on that event, the probability that the next-best

permitted alternative for the agent has agent utility no greater than  $z \leq u$  is

$$\frac{f(u)g(r(u), u)\phi'(x(z))}{f(u)g(r(u), u)\phi'(x(u))} = \frac{\phi'(x(z))}{\phi'(x(u))},$$

which has associated density  $\phi''(x(z))x'(z)/\phi'(x(u))$ . Therefore the indifference condition required for optimality is

$$r(u) + \alpha u = \frac{1}{\phi'(x(u))} \int_0^u [V(r(z), z) + \alpha z] \phi''(x(z)) x'(z) dz \quad (6)$$

for all  $u \in [0, u_{\max}]$ .

In particular, we see from (6) that  $r(0) = 0$ . This implies that the principal does not wish to restrict the desirable projects available to the agent whose best project has only zero payoff for her, i.e., there is “no distortion at the bottom”. The reason for this is that when  $u = 0$  there is no strategic benefit to restricting choice. (The strategic effect of raising  $r(u)$  above  $-\alpha u$  is to increase the probability that the agent will choose a smaller  $z$ , and this effect cannot operate when  $u = 0$ .) Differentiating (6) with respect to  $u$  and using (4) implies the Euler equation for the principal’s problem, which is expression (7) below.

**Proposition 1** *The principal’s problem of maximizing (3) subject to (4) and (5) over all piecewise-continuous threshold functions  $r(\cdot)$  has a solution. This solution is differentiable and satisfies the Euler equation*

$$r'(u) + \alpha = [V(r(u), u) - r(u)][1 - G(r(u), u)]f(u) \frac{\phi''(x(u))}{\phi'(x(u))} \quad (7)$$

with initial condition  $r(0) = 0$ . A sufficient condition for a threshold function which satisfies the Euler equation to be a global optimum is that

$$\zeta(x) \equiv \frac{\phi''(x)}{\phi'(x)} \text{ weakly decreases with } x. \quad (8)$$

**Proof.** The proofs of this and of subsequent Propositions are in the appendix. ■

Expression (7) reveals that  $\zeta$  in (8) is important for the form of the solution. A short list of examples for this term includes:

- If  $n$  is known to be exactly  $N \geq 1$  for sure (so  $q_N = 1$ ), then  $\phi(x) = x^N$  and  $\zeta(x) = (N - 1)/x$ .

- If  $n$  is Poisson with mean  $\mu$  (so  $q_n = e^{-\mu} \frac{\mu^n}{n!}$  for  $n \geq 0$ ) then  $\phi(x) = e^{-\mu(1-x)}$  and  $\zeta(x) \equiv \mu$ .
- If  $n$  is Binomial (the sum of  $N$  Bernoulli variables with success probability  $a$ ) then  $\phi(x) = (1 - a(1 - x))^N$  and  $\zeta(x) = a(N - 1)/(1 - a(1 - x))$ . The “known  $n$ ” case is a special case of the Binomial with  $a = 1$ . The Poisson is a limit case of the Binomial when  $aN = \mu$  and  $a \rightarrow 0$ .
- If  $n$  is Geometric (so  $q_n = (1 - a)a^{n-1}$  for  $n \geq 1$  and some parameter  $a \in (0, 1)$ ) then  $\phi(x) = (1 - a)x/(1 - ax)$  and  $\zeta(x) = 2a/(1 - ax)$ .

Assumption (8), which states that  $\phi'(x)$  is a log-concave function, is valid for the Binomial distribution—and hence for the “known  $n$ ” and Poisson sub-cases—but not for the Geometric distribution.

Define the “naive” threshold rule to be  $r_{naive}(u) = -\alpha u$ . This is the threshold rule which permits all desirable projects, i.e., those projects such that  $v + \alpha u \geq 0$ . This rule might be implemented by a principal who ignored the strategic effect that the agent will only choose the project with the highest  $u$  whenever she has a choice. As such, the naive rule is optimal when the agent never has a choice of project, i.e., when  $q_0 + q_1 = 1$ . (In this case  $\phi'' \equiv 0$ , the right-hand side of (7) vanishes, and so  $r(\cdot) \equiv r_{naive}(\cdot)$  is optimal.) Outside this case, though, the right-hand side of (7) is strictly positive. Since  $r'(u) + \alpha > 0$  and  $r(0) = 0$  it follows that  $r(u) > r_{naive}(u)$  when  $u > 0$ . Therefore, the principal forbids some strictly desirable projects (and never permits an undesirable project). Moreover, the gap between the optimal and the naive rule,  $r(u) - r_{naive}(u)$ , strictly increases. We state this formally as:

**Corollary 1** *Suppose the agent sometimes has a choice of project (i.e.,  $q_0 + q_1 < 1$ ). Then it is optimal for the principal to forbid some strictly desirable projects, and the gap between the optimal threshold rule  $r(u)$  and the naive threshold rule  $r_{naive}(u)$  widens with  $u$ . In particular, when  $\alpha = 0$  the optimal threshold rule increases with  $u$ .*

What is the intuition for why the principal wishes to exclude some desirable projects from the permitted set, whenever the agent sometimes has a choice of project? Suppose the principal initially allows all desirable projects, so that  $r(u) \equiv$

$r_{naive}(u)$ . If the principal increases  $r(\cdot)$  slightly at some  $u > 0$ , the direct cost is approximately zero, since the principal then excludes projects about which he is almost indifferent (since  $r(u) + \alpha u = 0$ ). But there is a strictly beneficial strategic effect: there is some chance that the agent’s highest- $u$  project is excluded by the modified permitted set, in which case there is a chance that she chooses another project which is permitted, say with  $z < u$ . This alternative project is unlikely to be marginal for the principal, and the principal will expect to get payoff  $V(r(z), z) + \alpha z$ , which is strictly positive when  $r(z) = -\alpha z$ . This argument indicates that it is beneficial to restrict desirable projects, and not to permit any undesirable projects. Moreover, it is intuitive that the strategic effect is more important for higher  $u$ , since it applies over a wider range  $z < u$ , and this explains why  $r(u) - r_{naive}(u)$  increases with  $u$ .

We next discuss some comparative statics for this problem.

**Proposition 2** *Let  $\alpha_L$  and  $\alpha_H$  be two weights placed by the principal on the agent’s payoff, where  $\alpha_L < \alpha_H$ . Let  $r_i(\cdot)$  and  $x_i(\cdot)$  solve the Euler equation (7) when  $\alpha = \alpha_i$  for  $i = L, H$ . If assumption (8) holds then  $x_L(0) \geq x_H(0)$ , i.e., the fraction of permitted projects increases with  $\alpha$ .*

Thus we see that the more the principal cares about the utility of the agent, the more discretion—in the sense of a greater fraction of projects being permitted—the agent is given.<sup>10</sup> This result is similar to the “ally principle” in the Holmstrom-type models mentioned in section 1, where the more likely the agent’s preferences were to be close to the principal’s, the more discretion the agent was given.

A second way in which the ally principle might be expected to be seen concerns the extent of correlation between  $u$  and  $v$ . Intuitively, when  $u$  is positively correlated with  $v$ , the agent’s incentives are likely to be aligned with those of the principal. In the limit of perfect positive correlation, since the agent’s best project is always the principal’s best project, it is optimal to give the agent complete freedom to choose a project. (By contrast, with strong negative correlation, the agent’s best permitted project is likely to be the principal’s worst permitted project, at least when  $\alpha$  is small.) However, it is not obvious how formally to define a notion of “more correlation” which

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<sup>10</sup>It is not necessarily the case that the threshold rules are *nested* so that  $r_H(\cdot) \leq r_L(\cdot)$ , and one can find examples where the two threshold rules cross for some positive  $u$ .

could be used as basis for general comparative statics analysis. Instead, in section 2.2 we discuss an example which confirms this intuition.

It is also intuitive that when the agent is likely to have more projects to choose from, the principal will further constrain the permitted set of projects. With more projects available, the agent is likely to have at least one which lies close to the principal’s preferred project. A notion of “more projects” which ensures that this intuition is valid is the familiar *monotone likelihood ratio property* (MLRP). The details are provided in the next result:

**Proposition 3** *Let  $(q_0^L, q_1^L, q_2^L, \dots)$  and  $(q_0^H, q_1^H, q_2^H, \dots)$  describe two probability distributions for the number of projects which satisfy MLRP, i.e.,  $q_n^H/q_n^L$  weakly increases with  $n$ . Let  $\phi_i(\cdot)$  be the PGF corresponding to  $(q_0^i, q_1^i, q_2^i, \dots)$ , and suppose  $\phi_L(x)$  satisfies (8). Let  $r_i(\cdot)$  and  $x_i(\cdot)$  solve the Euler equation (7) when the number of projects is governed by  $(q_0^i, q_1^i, q_2^i, \dots)$  for  $i = L, H$ . Then  $x_H(0) \geq x_L(0)$ .*

Thus, the fraction of permitted projects falls when the agent is likely to have more projects available.<sup>11</sup> The requirement that the number of projects is ordered by MLRP is a stronger requirement than first-order stochastic dominance. Indeed, there are examples where stochastic dominance leads to a *smaller* fraction of projects being excluded.<sup>12</sup> Moreover, it is not necessarily the case that the principal benefits when the agent has access to more projects. When there is strong negative correlation between  $u$  and  $v$ , an agent choosing from more projects is likely, all else equal, to choose a worse project from the principal’s perspective.<sup>13</sup>

<sup>11</sup>As emphasized in Lyons (2002), in the merger application it is more likely that the more stringent consumer standard is superior to a total welfare standard in large, complex economies where merger possibilities may be more plentiful.

<sup>12</sup>An example where adding more projects widens the optimal set of permitted projects is the following. Suppose initially the agent has no projects at all with probability  $1 - \varepsilon$  and exactly two projects with probability  $\varepsilon$ . Because the state when no projects materialize plays no role in the determination of  $r(\cdot)$ , the optimal threshold rule for this agent is just as if there were two projects for sure. Such a threshold rule will strictly exclude some desirable projects. Consider next the situation in which the agent has exactly one more project than the previous situation (i.e.,  $n = 1$  with probability  $1 - \varepsilon$  and  $n = 3$  with probability  $\varepsilon$ ). Whenever  $\varepsilon$  is small, the state where there is only one project will dominate the choice of  $r(\cdot)$ , and almost all desirable projects will be permitted, thus widening the set of permitted projects. One can check that this pair of probability distributions does not satisfy MLRP.

<sup>13</sup>Our benchmark model assumes that each realization of project characteristics  $(u, v)$  is independent across projects. If instead project characteristics were positively correlated across projects, it is plausible that the effect of correlation would be similar to having fewer independent projects. (For instance, in the extreme case where all projects had the same realization of  $(u, v)$ , the situation is just as if the agent had a single project to choose from, in which case the naive rule is optimal.)

Without making further assumptions, it is hard to make more progress in characterizing the solution to (7). In the remainder of section 2 we examine further properties of the solution in three special cases.

## 2.1 Independent payoffs and $\alpha = 0$

Suppose that the distribution of  $v$  is independent of  $u$  and that  $\alpha = 0$ . Then the principal does not care about the agent's choice of  $u$ , either directly (since  $\alpha = 0$ ) or indirectly since the distribution of his payoff  $v$  does not depend on  $u$ .

Write  $G(v)$  and  $V(r)$  as functions which do not depend on  $u$ . It follows that

$$\begin{aligned} \frac{d}{du} \left[ \frac{V(r(u)) - r(u)}{f(u)} \frac{d}{du} \phi(x(u)) \right] &= \frac{d}{du} [(V(r(u)) - r(u))(1 - G(r(u)))\phi'(x(u))] \\ &= \frac{d}{du} \left[ \left( \int_{r(u)}^{v_{\max}} (1 - G(v)) dv \right) \phi'(x(u)) \right] \\ &= -r'(u)(1 - G(r(u)))\phi'(x(u)) + (V(r(u)) - r(u))(1 - G(r(u)))^2 f(u)\phi''(x(u)) \end{aligned}$$

which equals zero at the optimum from (7). Therefore, the second-order Euler equation reduces to the first-order equation

$$[V(r(u)) - r(u)] \frac{d}{du} \phi(x(u)) = kf(u) \quad (9)$$

for some positive constant  $k$ .

It follows that the principal obtains the same expected payoff with all density functions  $f(\cdot)$  for  $u$ . To see this, change variables in (9) from  $u$  to  $F(u)$ . That is to say, write  $\hat{r}(F(u)) \equiv r(u)$  and  $\hat{x}(F(u)) \equiv x(u)$ , so that  $\hat{r}$  represents the threshold rule expressed in terms of the cumulative *fraction* of  $u$ -projects  $F$ . Then (9) becomes

$$[V(\hat{r}(F)) - \hat{r}(F)] \frac{d}{dF} \phi(\hat{x}(F)) \equiv k, \quad (10)$$

with endpoint conditions  $\hat{r}(0) = 0$  and  $\hat{x}(1) = 1$ . Here, the optimal threshold rule  $\hat{r}(\cdot)$  does not depend on the distribution for  $u$ , as long as  $u$  is continuously distributed.<sup>14</sup> As such, only *ordinal* rankings of  $u$  matter for the principal in this case.

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<sup>14</sup>Note that this argument requires us to change variables in expression (9), and so  $F(u)$  needs to be differentiable and, in particular, the distribution for  $u$  has no "atoms". If there were atoms, then we would need to consider what project the agent would choose in the event of a tie, when there were two projects which yielded the same maximal agent payoff  $u$ .

## 2.2 Exponential distribution for $v$

Suppose next that  $v$  given  $u$  is exponentially distributed on  $[0, \infty)$  with mean  $\lambda(u)$ , so that  $G(v, u) = 1 - e^{-v/\lambda(u)}$ . Suppose that  $\alpha = 0$ . Since  $V(r, u) \equiv r + \lambda(u)$  in this example, the Euler equation (7) is

$$r'(u) = \lambda(u) \frac{d}{du} \log \phi'(x(u))$$

with initial condition  $r(0) = 0$ . Since we wish to compare policies across different distributions for  $(u, v)$ , the threshold rule  $r$  is not in itself insightful. Rather, we study the *fraction* of permitted type- $u$  projects, and given  $r$  write  $p(u) = 1 - G(r(u), u) = e^{-r(u)/\lambda(u)}$  for this fraction. Writing the Euler equation in terms of  $p$  rather than  $r$  implies that

$$\frac{d}{du} \log p(u) = - \left[ \frac{d}{du} \log \phi'(x(u)) + \frac{\lambda'(u)}{\lambda(u)} \log p(u) \right] \quad (11)$$

with initial condition  $p(0) = 1$ .

Consider first the case where  $\lambda$  is constant, so that  $u$  and  $v$  are independent. Expression (11) implies that  $p\phi'(x)$  does not vary with  $u$ , and it follows that  $\phi(x(u)) = k_1 F(u) + k_2$  for constants  $k_1$  and  $k_2$ . Since  $\phi(x(u_{\max})) = 1$ , it follows that  $k_1 + k_2 = 1$ . Since  $p(0) = 1$ , it follows that  $k_1 = \phi'(x_0)$ , where  $x_0 = x(0)$  is the fraction of banned projects at the optimum when  $u$  and  $v$  are independent. In sum, at the optimum  $x(\cdot)$  satisfies  $\phi(x(u)) = 1 - \phi'(x_0)(1 - F(u))$ . Evaluating this at  $u = 0$  implies that the fraction of banned projects is the unique solution to

$$\phi(x_0) + \phi'(x_0) = 1 . \quad (12)$$

Next, suppose that  $u$  and  $v$  are positively correlated in the (strong) sense that  $\lambda(u)$  increases with  $u$ . Write  $h(u) \equiv p(u)\phi'(x(u))$ , which from (11) is an increasing function. It follows that

$$\phi(x(u)) = 1 - \int_u^{u_{\max}} h(\tilde{u}) f(\tilde{u}) d\tilde{u} = 1 - h(0)(1 - F(u)) - \int_u^{u_{\max}} [h(\tilde{u}) - h(0)] f(\tilde{u}) d\tilde{u} .$$

Since  $h(0) = \phi'(\tilde{x}_0)$ , where  $\tilde{x}_0$  denotes the fraction of banned projects in this case with positive correlation, and  $h$  is increasing, it follows that  $\phi(\tilde{x}_0) + \phi'(\tilde{x}_0) < 1$ . Since  $\phi(\cdot) + \phi'(\cdot)$  is an increasing function, it follows that the fraction of permitted projects is higher with positive correlation than with independence. A parallel argument establishes that when there is negative correlation, in the sense that  $\lambda$  decreases

with  $u$ , the fraction of permitted projects is smaller than with independence. In this exponential example, then, positive correlation between  $u$  and  $v$  is associated with a greater number of permitted projects than negative correlation.

### 2.3 Poisson distribution for the number of projects

As our third special case suppose that the number of projects follows a Poisson distribution with mean  $\mu$ , in which case the Euler equation (7) reduces to a first-order differential equation in  $r(u)$ :

$$r'(u) + \alpha = \mu[V(r(u), u) - r(u)][1 - G(r(u), u)]f(u) ; r(0) = 0 . \quad (13)$$

The next result shows that the comparative statics of  $r(\cdot)$  with respect to  $\alpha$  and  $\mu$  are stronger than the corresponding results in the general setting reported above in Propositions 2 and 3.

**Proposition 4** *With a Poisson distribution for the number of available projects, the optimal threshold rule  $r(\cdot)$  is pointwise decreasing in  $\alpha$  and increasing in  $\mu$ .*

To obtain some explicit solutions for the threshold rule, suppose that  $(u, v)$  is uniformly distributed on the rectangle  $[0, 1] \times [-1, 1]$ . In this case, (13) becomes the homogeneous equation

$$r'(u) = \frac{1}{4}\mu(1 - r(u))^2 - \alpha ; r(0) = 0 . \quad (14)$$

Note that if  $\mu = 4\alpha$  then the solution to (14) is simply the flat rule  $r(u) \equiv 0$ . Thus, in the merger context, if the regulator wishes to maximize total welfare (so  $\alpha = 1$ ), then when the expected number of merger possibilities is  $\mu = 4$  the regulator should optimally enforce a consumer welfare standard.

The solution to (14) when  $\mu \neq 4\alpha$  is given implicitly by

$$\int_0^{r(u)} \frac{1}{\frac{1}{4}\mu(1 - r)^2 - \alpha} dr = u . \quad (15)$$

When  $\alpha = 0$ , expression (15) yields the simple formula

$$r(u) = \frac{\mu u}{4 + \mu u} . \quad (16)$$

When  $\alpha > 0$  expression (15) can be integrated using partial fractions to give

$$r(u) = \left(1 - \frac{4\alpha}{\mu}\right) \frac{e^{u\sqrt{\alpha\mu}} - 1}{(1 + \sqrt{4\alpha/\mu})e^{u\sqrt{\alpha\mu}} - (1 - \sqrt{4\alpha/\mu})}. \quad (17)$$

Figure 3 plots the rule (17) for  $\alpha = 1$  and various  $\mu$ . Here, higher curves correspond to higher  $\mu$  as in Proposition 4. The straight line depicted for  $\mu = 0$  is just the naive rule which permits any desirable project.

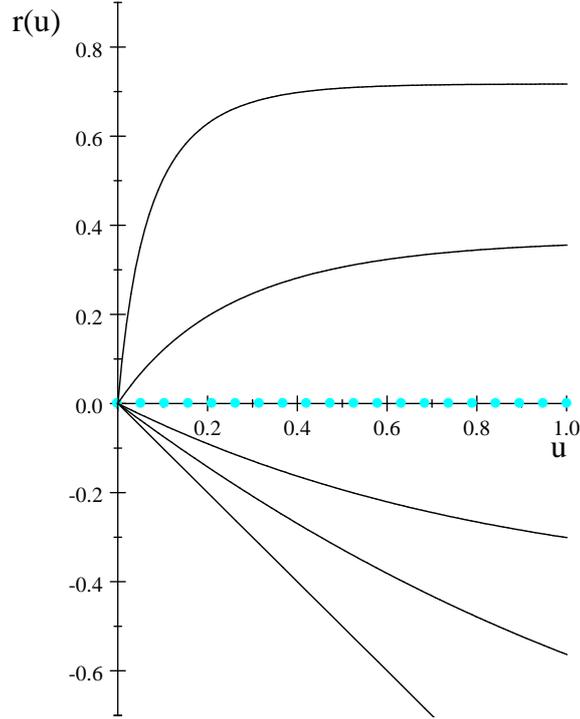


Figure 3: Uniform-Poisson case with  $\alpha = 1$  and  $\mu = 0, 1, 2, 4$  (dotted), 10 and 50

A final observation about the Poisson distribution concerns the principal's expected payoff, which from (6) evaluated at  $u_{\max}$  is equal to  $r(u_{\max}) + \alpha u_{\max}$ . (Recall that the density of the agent's choice of  $u$  is  $\frac{d}{du}\phi(x(u))$  and that the Poisson case entails  $\phi''(x) \equiv \mu\phi'(x)$ .) For instance, in the uniform example with  $\alpha = 0$  where the threshold rule is given by (16), it follows that the Principal's maximum expected payoff is  $r(1) = \mu/(4 + \mu)$ .

## 3 Variants of the Benchmark Model

### 3.1 Incentives to find a project

The benchmark model in section 2 assumed that the number of projects was exogenous to the agent. In such a framework the agent does not need to be given an incentive to discover projects. In this variant we suppose that the agent needs to exert effort to find a project. We do this in the simplest possible way, so that by exerting effort  $e$  the agent finds a single project with probability  $e$ , while with remaining probability  $1 - e$  no project emerges.<sup>15</sup> If she finds a project, that project's characteristics  $(u, v)$  are realized according to the same density functions  $f$  and  $g$  as in the benchmark model. To achieve success probability  $e$  the agent incurs the private cost  $c(e)$ . Here,  $c(\cdot)$  is assumed to be convex, with  $c(0) = c'(0) = 0$  and  $c'(1) = \infty$ .

Since the agent's effort incentives depend on her *expected* payoff across all permitted projects, her attitude towards risk is relevant, and in this section we assume the agent is risk neutral. The principal's payoff is a weighted sum of the agent's payoff (including her cost of effort) and the expected value of  $v$ , where the weight on the agent's payoff by the principal is  $\alpha$ . The principal determines a piecewise-continuous function  $r(\cdot)$  such that any  $(u, v)$  project with  $v \geq r(u)$  is permitted.

If she discovers a project, the agent's expected payoff excluding effort cost is

$$A = \int_0^{u_{\max}} u[1 - G(r(u), u)]f(u) du, \quad (18)$$

and the agent will choose effort  $e$  to maximize her net payoff  $eA - c(e)$ . Clearly, a reduction in the threshold rule  $r(\cdot)$  induces a higher value of  $A$  in (18), which in turn leads unambiguously to greater effort from the agent.<sup>16</sup> Since high effort benefits the principal as well as the agent, the principal has a reason (beyond the weight  $\alpha$  placed on the agent's interests) to increase the leeway given to the agent.

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<sup>15</sup>A richer model would involve the agent being able to affect the expected number of projects, so that the agent may end up with a choice of project. (For instance, if the number of projects follows a Poisson distribution, the agent could choose  $\mu$  by incurring cost  $C(\mu)$ , say.) The principal's optimal policy in this situation has some similarities to the policy when the number of projects was exogenous: the threshold rule is nonlinear and involves  $r(0) = 0$ . However, like the model of costly discovery analyzed in this section, the threshold rule reflects the need to give the agent an incentive to find more projects (which typically benefits the principal as well as the agent), and it may be optimal *ex ante* to permit projects which are undesirable *ex post*.

<sup>16</sup>This is akin to the "initiative effect" of delegation in Aghion and Tirole (1997).

If we write

$$\sigma(A) \equiv \max_{e \geq 0} : eA - c(e)$$

for the agent's maximum payoff given  $A$ , then  $\sigma$  is a convex increasing function and  $\sigma'(A)$  is the agent's choice of effort  $e$  given her reward  $A$ . The principal chooses  $r(\cdot)$  to maximize his expected payoff

$$\alpha\sigma(A) + B\sigma'(A) , \quad (19)$$

where

$$B = \int_0^{u_{\max}} V(r(u), u)[1 - G(r(u), u)]f(u) du .$$

The principal's optimal policy is described in the next result:

**Proposition 5** *The principal's optimal policy takes the form*

$$r(u) + \left[ B \frac{\sigma''(A)}{\sigma'(A)} + \alpha \right] u \equiv 0 . \quad (20)$$

Thus, the optimal threshold rule is a ray emanating from the origin. This ray is downward sloping and weakly steeper than the principal's naive rule,  $r_{naive}(u) \equiv -\alpha u$ . The only situation in which the principal implements his naive rule is when  $\sigma'' = 0$ , which applies when the agent's success probability does not respond to incentives, i.e., there is an *exogenous* success probability  $e$ . Outside this case, though, the principal allows some projects which are strictly undesirable ( $v + \alpha u < 0$ ) in order to stimulate the agent's effort.<sup>17</sup> The more that the agent responds to incentives (in the sense that the function  $\sigma''(\cdot)/\sigma'(\cdot)$  is shifted upwards), the more leeway she should have to choose a project. This distortion is the opposite to the bias in the "choosing" model in section 2, where the principal forbade some desirable projects.

Some intuition for the linearity of  $r(u)$  comes from the following argument. The principal's payoff in (19) is a function of both  $A$  (the expected value of  $v$  from a single project given that the project is only implemented if it is permitted) and  $B$  (the expected value of  $u$  from a project given that the project is only implemented if it is permitted), and  $A$  and  $B$  are in turn functions of the principal's permission rule

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<sup>17</sup>This feature is also seen in Aghion and Tirole (1997) and Baker, Gibbons, and Murphy (1999). By contrast, as discussed in section 1, Szalay (2005) presents a model where information gathering incentives are enhanced by forbidding projects which both the principal and agent might often wish to implement.

$r(\cdot)$ . The problem of choosing  $r(\cdot)$  to maximize a (nonlinear) function of  $A$  and  $B$  has the same first-order condition as maximizing a linear sum  $A + \gamma B$  for some constant  $\gamma$ . That is to say, the solution to the principal's problem is obtained by choosing  $r(\cdot)$  to maximize

$$\int_0^{u_{\max}} \int_{r(u)}^{v_{\max}} [v + \gamma u] g(v, u) f(u) \, dv du$$

for some constant  $\gamma$ , the solution to which is clearly to set  $r(u) = -\gamma u$  so that only the positive  $[v + \gamma u]$  are contained in the integral.

In earlier work we analyzed a more complicated version of this problem in which the agent searches sequentially for a satisfactory project, and can influence the arrival rate of new projects by incurring effort. Then the agent might not implement the first permitted project which emerges, but rather wait until she finds a permitted project which achieves a reservation utility, where this reservation utility will depend on the threshold rule  $r(\cdot)$  as well as her discount rate. A linear threshold rule is again optimal for the principal, although not necessarily a rule which starts at the origin. When the principal and agent are more impatient, the threshold rule is shifted downwards, so that the principal is willing to accept a less good project, and with less delay. In the limit of extreme impatience, the dynamic search problem essentially reduces to the framework discussed in this section where the agent tries to discover a single project.

### 3.2 Paying for a good project

Most of our analysis presumes that monetary incentives to choose a desirable project are not available or desirable, for reasons outside the model. In this second variant we briefly discuss the principal's optimal policy when he can condition the agent's payment on her performance. We will see that, even within the confines of the model, monetary incentives are not always desirable.

First, suppose that the agent is risk-neutral and is able to bear large losses *ex post*. As in most principal-agent models, the principal here is able to attain his first-best outcome with the use of monetary incentives. The first-best outcome is obtained when (i) he does not restrict the agent's choice of project, (ii) he pays the agent  $v$  when a type- $(u, v)$  project is implemented (and allows the agent to keep her benefit  $u$ ), and (iii) extracts the agent's entire expected surplus from this scheme in the form

of a payment to the principal up front. Such a scheme is akin to “selling the firm” to the agent, and gives the agent ideal incentives to choose the best available project while leaving the agent with zero expected rent.

Outside this extreme case, however, the first-best will not be attainable, and there may again be a role for restricting the agent’s discretion. Moreover, the use of monetary incentives will not always be optimal for the principal.<sup>18</sup> To illustrate most simply, consider the situation in which the agent is liquidity constrained in the sense that she must receive a non-negative salary (excluding her payoff  $u$  from an implemented project) in all outcomes.<sup>19</sup> For simplicity, suppose that there are two possible kinds of project, one of which is preferred by the agent while the other is preferred by the principal. Specifically, the “bad project” has payoffs  $(u_H, v_L)$  and the “good project” has payoffs  $(u_L, v_H)$ , where  $0 < u_L < u_H$  and  $0 < v_L < v_H$ .<sup>20</sup> Write  $\Delta_u = u_H - u_L$  and  $\Delta_v = v_H - v_L$  and suppose  $\Delta_v > \Delta_u$  so that  $(u_L, v_H)$  is indeed socially the good project. The difference  $\Delta_u$  can be interpreted as the “bias” of the agent. Since there are just two types of project, what matters is the probability that the agent has only the good project available (denoted  $P_G$ ), the probability she has only the bad project (denoted  $P_B$ ), and the probability she has a choice of project types (denoted  $P_{GB}$ ).

If the principal bans the bad project, his payoff is

$$\pi_1 = (P_G + P_{GB})(v_H + \alpha u_L) .$$

If the principal allows both projects but does not use monetary rewards, his payoff is

$$\pi_2 = P_G(v_H + \alpha u_L) + (P_{GB} + P_B)(v_L + \alpha u_H) .$$

(Here, the agent will choose the bad project whenever that project is available.) The remaining policy is to give the agent a monetary incentive equal to  $\Delta_u$  to choose the

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<sup>18</sup>For instance, if there are very many projects available to the agent, the first-best is approximately achieved by permitting the agent to choose only the best projects for the principal and making no monetary payments to the agent.

<sup>19</sup>A similar restriction to non-negative payments is made in Aghion and Tirole (1997), Berkovitch and Israel (2004), and Alonso and Matouschek (2008, section 8.1).

<sup>20</sup>If  $u_L < 0$  then without monetary compensation the agent will not reveal the good project, even when that is the only option available to her. In this case, the use of money rewards enables the good project to be implemented when available, which is an important benefit of using money rewards relative to restricting the agent’s choice.

good project (and not to fetter her discretion), which entails payoff

$$\pi_3 = (P_G + P_{GB})(v_H - \Delta_u + \alpha u_H) + P_B(v_L + \alpha u_H) .$$

(Here, the agent will implement the good project whenever such a project is available.)

We require  $\pi_3 \geq \max\{\pi_1, \pi_2\}$  in order for monetary incentives to be optimal. Now

$$\pi_3 - \pi_1 > 0 \Leftrightarrow P_B(v_L + \alpha u_H) > (P_G + P_{GB})(1 - \alpha)\Delta_u$$

and

$$\pi_3 - \pi_2 > 0 \Leftrightarrow P_{GB}(\Delta_v - \Delta_u) > P_G(1 - \alpha)\Delta_u .$$

These inequalities are jointly satisfied when the agent's bias  $\Delta_u$  is sufficiently small (i.e., when little money needs to be paid to change agent behaviour) or when  $\alpha$  is close to 1 (so that payments to the agent are not costly for the principal). By contrast, monetary incentives should not be used when the agent's bias  $\Delta_u$  is large or  $v_L$  is small. Finally, note that increasing the number of projects draws will make it more likely the the agent has a choice of project types, so that  $P_{BG}$  rises, and this makes it less likely that  $\pi_3 > \pi_1$ .<sup>21</sup> Thus, all else equal we expect that a greater number of project opportunities, or a larger agent bias, will make the use of money rewards less attractive.<sup>22</sup>

Berkovitch and Israel (2004) have analyzed a related model, also with binary project types. Provided the manager's bias is not too large, they show [Proposition 1(a)] that (i) if the "good" (i.e., less capital intensive) project is relatively likely to be available, then the optimal policy is (stochastically) to ban the bad project rather than to be reward the manager when she brings forward a good project, and (ii) if a good project is less likely to emerge it becomes optimal to permit both projects but to pay the agent for a good project.

This example illustrates a more general trade-off between banning mediocre projects and rewarding the choice of good ones. When he bans mediocre projects the principal suffers the cost that such projects are not implemented when they are the only ones available. Rewarding the choice of good projects avoids this cost, but instead involves

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<sup>21</sup>For instance, if there  $N$  project opportunities for sure, and each opportunity has probability  $P$  of yielding a bad project, then  $P_B = P^N$ ,  $P_G = (1 - P)^N$ , and  $P_{GB} = 1 - P^N - (1 - P)^N$ .

<sup>22</sup>See Figure 9 in Alonso and Matouschek (2008) for an illustration in their framework of the limited gains to the principal in being able to make contingent payments to the agent rather than just banning projects.

paying the reward whenever at least one good project is available. Restricting choice is therefore preferred when the chance of having only mediocre projects is small, which is more likely to be true when the agent can choose from many projects.<sup>23</sup> In richer settings than the illustrative binary example above, it may be optimal both to ban mediocre projects and reward the choice of good projects. In addition, if the agent is not liquidity constrained, it is possible to financially penalize her choice of bad projects, which could well be preferable to an outright ban. We leave a more complete analysis of the interactions between restricting choice and monetary incentives as a topic for further work.

### 3.3 A more complex delegation scheme

Our benchmark scheme simply involves specifying a set of permitted projects, and the agent chooses her preferred available project in this set. In particular, only the agent’s chosen project is subject to verification. If however the principal could easily determine the genuine feasibility of all reported projects (so there are no significant costs of auditing reported but unchosen projects), the principal may be able to do better by inducing the agent to list more than one project. Since the listed projects have characteristics which can be verified by the principal, the agent can only reveal true projects. But, except in the implausible case in which the number of available projects is known in advance (the “known  $n$ ” case), the agent need only report those projects she wishes to report and she cannot be made to reveal the “whole truth”.<sup>24</sup> In such cases, the most general (deterministic) delegation scheme takes the following form: if the agent reveals a list of feasible projects, the principal picks (in a pre-determined way) one of these projects, or implements no project.<sup>25</sup>

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<sup>23</sup>Another reason why monetary incentives are not always given to an agent is that the agent performs several tasks, and giving incentives to do one task well might induce the agent to underperform on other, unmeasured, aspects of her job (see Holmstrom and Milgrom (1991)).

<sup>24</sup>This information structure is akin to games of persuasion. For instance, consider the signalling model of Shin (2003) in which a number of projects are undertaken by a firm, the sum of whose outcomes determines the returns to shareholders. (Therefore, unlike our framework, the firm’s manager does not choose *which* project to pursue.) But the manager has interim information about the outcome of a random subset of the projects, and she can reveal a subset of those project outcomes she knows. Thus, the manager can only conceal poor outcomes, not make up good ones. One plausible equilibrium is where the manager reveals all the good news and conceals all bad news.

<sup>25</sup>An example of such a scheme concerns work-related travel plans. An employee may be able to get permission for an expensive flight more easily if she reveals a number of other expensive quotes than if she just provides one. As we discuss further below, such a scheme may give the principal some

For simplicity, we analyze this issue in the context of the binary project types discussed in the previous section. In this context the most general (deterministic) delegation scheme involves the agent reporting a list of projects as summarized by the pair of integers  $(b, g)$ , where  $b$  is her reported number of bad projects and  $g$  her reported number of good projects. The constraint that the agent must tell the truth, but not necessarily the whole truth, is captured by the requirement that the agent's reports satisfy  $b \leq B$  and  $g \leq G$ , where  $B$  and  $G$  are the actual numbers of bad and good projects. A delegation scheme in general is a choice function which maps a report  $(b, g)$  into a decision to implement (i) either a good project (provided  $g \geq 1$ ), (ii) a bad project (provided  $b \geq 1$ ), or (iii) no project at all. However, the principal need only consider a particular family of delegation schemes:

**Lemma 2** *The principal can restrict attention to the following family of delegation schemes: if the agent reports she has at least one good project, that project is implemented; if the agent reports she has  $b$  bad projects (and no good projects), a bad project is implemented provided that  $b \geq m$  for some (possibly infinite) integer  $m$ .*

**Proof.** Suppose, contrary to the claim, that for some report  $(b, g)$  with  $g \geq 1$  the principal either implements a bad project (if  $b \geq 1$ ) or nothing at all. If instead the principal modifies his policy so that with this report  $(b, g)$  he now chooses to implement a good project, then this modification weakly increases the principal's payoff. (If the modification does not alter the agent's report, it clearly boosts the principal's payoff. If it does induce the agent to change her report, the only way the principal's payoff could be lowered is if the original policy with report  $(b, g)$  was to implement a bad project, and now the agent switches to a report which causes no project to be implemented. However, it is clear that it cannot be in the agent's interest to switch to such a report, since she could obtain  $u_L > 0$  from the new policy by making her original report.) Therefore, it remains to describe the principal's policy when the agent reports only a number of bad projects. But since the agent can always reduce the number of bad projects she reports, all that matters is the *minimum* number of bad projects reported, say  $m$ , which ensures that a bad project is implemented. This completes the proof. ■

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assurance that the employee is not concealing a cheap but personally inconvenient flight option.

Thus, the principal need consider only the single number  $m$ , the minimal number of bad realizations needed to authorize a bad project, when choosing his preferred scheme.<sup>26</sup> When  $m = 1$  the agent can implement any project she chooses and when  $m = \infty$  there is a blanket ban on bad projects, and these are the two possible policies in the benchmark model when only the implemented project's characteristics could be verified. Note that if the maximum possible number of projects is  $N < \infty$ , a delegation scheme with  $m = N$  is surely superior to a blanket ban on bad projects, since it allows a desirable project to be implemented when there are  $N$  projects and all are bad, and it achieves the same outcome otherwise. (Indeed, if the number of projects is known to be  $N$  for sure, setting  $m = N$  yields the first-best outcome for the principal.)

We next calculate the optimal choice for  $m$ . Suppose a given project has probability  $P$  of being bad, and the PGF for the number of project draws is  $\phi(\cdot)$ . Then the number of bad projects has PGF  $\phi(1 - P + Px)$ .<sup>27</sup> Therefore, the probability there are exactly  $n$  bad projects is

$$\frac{P^n}{n!} \phi^{[n]}(1 - P),$$

where  $\phi^{[n]}$  is the  $n$ th derivative of  $\phi$ .<sup>28</sup> The probability there are exactly  $n$  bad projects and no good projects is  $q_n P^n = \frac{P^n}{n!} \phi^{[n]}(0)$ , and so the probability there are exactly  $n$  bad projects and at least one good project is  $\frac{P^n}{n!} [\phi^{[n]}(1 - P) - \phi^{[n]}(0)]$ . It follows that the principal's expected payoff with threshold  $m \geq 1$ , denoted  $W_m$ , is

$$W_m = (v_H + \alpha u_L) \sum_{n=0}^{m-1} \frac{P^n}{n!} [\phi^{[n]}(1 - P) - \phi^{[n]}(0)] + (v_L + \alpha u_H) \sum_{n=m}^{\infty} \frac{P^n}{n!} \phi^{[n]}(1 - P).$$

To understand this expression, note that when the agent has fewer than  $m$  bad projects and at least one good project, she has no choice but to implement a good

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<sup>26</sup>Green and Laffont (1986) analyze in general terms a principal-agent model where the agent's feasible reports depend on her private information. Our set-up in which the agent can conceal but not fabricate projects is a special case of their model, and one which satisfies their "nested range condition". Therefore, we can invoke their Proposition 1 to deduce that the principal can restrict attention to choice rules which induce the agent to report *all* her projects (in contrast to the rules in Lemma 2).

<sup>27</sup>In general, if  $m$  is a discrete random variable with PGF  $\phi_M$ , then the random variable generated by the sum of  $n$  independent realizations of  $m$ , where  $n$  is itself a discrete random variable with PGF  $\phi_N$ , has PGF  $\phi_N(\phi_M(\cdot))$ .

<sup>28</sup>Recall that for an arbitrary PGF  $\psi(x)$ , the probability of having realization  $n$  is equal to the coefficient of  $x^n$  in  $\psi$ , i.e., is equal to  $\psi^{[n]}(0)/n!$ .

project, which accounts for the first sum above, whereas if she has at least  $m$  bad projects she will implement a bad project, yielding the second sum above.

Note that  $W_{m+1} - W_m$  has the sign of

$$\frac{\Delta_v - \alpha\Delta_u}{v_H + \alpha u_L} - \frac{\phi^{[m]}(0)}{\phi^{[m]}(1 - P)}, \quad (21)$$

and so  $W_m$  is single-peaked in  $m$  provided that  $\phi^{[m]}(0)/\phi^{[m]}(1 - P)$  weakly increases with  $m$ , which in turn holds whenever  $\phi^{[m+1]}(x)/\phi^{[m]}(x)$  weakly decreases with  $x$  for each  $m \geq 1$ . (This condition is a stronger version of assumption (8) used in the benchmark model.) When  $W_m$  is single-peaked, the principal's optimal policy is to choose the smallest  $m$  such that (21) is negative. To illustrate, consider the case where the number of projects follows a Binomial distribution, i.e., the sum of  $N$  Bernoulli variables with success probability  $a$ , so that  $\phi(x) = (1 - a(1 - x))^N$ . Then for  $m \leq N$  expression (21) becomes

$$\frac{\Delta_v - \alpha\Delta_u}{v_H + \alpha u_L} - \left( \frac{1 - a}{1 - aP} \right)^{N-m}$$

which is indeed decreasing in  $m$ , and so the optimal  $m$  is the smallest  $m$  which makes the expression negative.<sup>29</sup> Notice that the optimal scheme is more permissive—in the sense that  $m$  is smaller—when  $\alpha$  is larger and when  $N$  is smaller, which parallels the comparative statics for the benchmark model in Propositions 2 and 3.

What about cases with an unbounded number of potential projects? When the number of projects comes from a Poisson distribution with mean  $\mu$ , expression (21) becomes

$$\frac{\Delta_v - \alpha\Delta_u}{v_H + \alpha u_L} - e^{-\mu(1-P)},$$

which is independent of  $m$ . Thus,  $W_m$  is monotonically increasing in  $m$  if the above expression is positive, in which case a blanket ban on bad projects is optimal. Alternatively,  $W_m$  decreases with  $m$  if the above expression is negative, in which case the agent should have authority to implement any project she chooses. In either event, in the Poisson case the more complicated delegation schemes considered in this section cannot improve on a simple scheme in which the agent is just presented with a set of permitted projects.

<sup>29</sup>For instance, if  $a = P = (\Delta_v - \alpha\Delta_u)/(v_H + \alpha u_L) = 1/2$ , then  $m = N - 1$  is optimal.

The intuition for this result comes from noting that in the Poisson case the number of good projects and the number of bad projects are themselves *independent* Poisson random variables (with respective means  $(1 - P)\mu$  and  $P\mu$ ). Thus, even if the principal could costlessly observe the number of bad projects, the realized number of bad projects has no impact on his decision about whether or not to permit bad projects. Since the complicated schemes in this section are a *costly* way to gain information about the number of bad projects (since the agent then sometimes implements a bad project when she has a good project), any such scheme must strictly underperform relative to the best simple scheme. This has the additional implication that in the Poisson case the principal’s optimal rule—say, to ban the bad project—is “renegotiation proof”, in the following sense: even if the agent can credibly reveal a number of projects which are not permitted, the principal has no incentive to adjust his permission set.

## 4 Conclusions

Proceeding from the motivating example of welfare standards in merger policy, we have explored the nature of optimal discretion for a principal to give to an agent when the agent may have a choice of project. The principal’s problem is to design the optimal set of permitted projects without knowing which projects are available to the agent, though being able to verify the characteristics of the project chosen by the agent. In other words, the problem is to set the optimal rule that the agent must obey, in circumstances where the principal can just check whether or not the rule has been met.

In the benchmark model the agent has a number (unknown to the principal) of projects to choose from. The optimal permission set excludes some projects that are desirable for the principal because the loss from excluding marginally desirable projects is outweighed by the expected gain from thereby inducing the choice of better projects. We showed (i) the principal permits more types of project when he puts more weight on the agent’s welfare, and (ii) the principal permits fewer types of project when the agent has more projects to choose from.

In one variant of this model, we supposed that by incurring a private cost the agent makes it more likely that a project emerges, and the optimal permission set

was characterised by a linear relationship between the payoffs of principal and agent. In order to encourage agent initiative, the principal permits some projects which are undesirable *ex post*, in contrast to the bias induced in the benchmark model. In a second variant, the principal was able to offer a monetary reward to the agent for choosing a good project, but with liquidity constraints on the agent it might nevertheless be preferable to ban mediocre projects than to reward good ones: the former policy has costs when all available projects are mediocre, while the latter involves payments whenever there is at least one good project. In a final variant, we consider a situation in which the principal can verify a *list* of reported projects. In some cases, the principal does better by using a more complex delegation scheme which, for instance, is more permissive towards mediocre projects when the agent she has several such projects. When the number of projects comes from a Poisson distribution, however, there is no gain in fine-tuning schemes in this manner.

It would be useful to examine more systematically than we do here the relative benefits of offering financial inducements (including penalties as well as rewards) to choose good projects versus banning mediocre projects. Another way to develop the analysis could be to multi-agent settings: it is after all a feature of many rules that they apply without discrimination to various agents in various situations.

## APPENDIX

**Proof of Proposition 1:** The principal's aim is to maximize (3) subject to the endpoint condition (5) and equation of motion (4). We consider the control variable  $r(\cdot)$  to be taken from the set of piecewise continuous functions defined on  $[0, u_{\max}]$  which take values in  $[v_{\min}, v_{\max}]$ , in which case  $x(\cdot)$  is continuous and piecewise differentiable.

Although this is already a well-posed optimal control problem, it is more convenient to consider  $s(u) \equiv \phi(x(u))$ , rather than  $x(u)$ , as the state variable. In this case the equation of motion (4) becomes

$$s'(u) = (1 - G(r(u), u))f(u)\tau(s(u)) , \tag{22}$$

where  $\tau(\cdot)$  is the function derived from  $\phi(\cdot)$  such that  $\phi'(x) \equiv \tau(\phi(x))$  for all  $0 \leq x \leq 1$ . (That is to say,  $\tau(s) \equiv \phi'(\phi^{-1}(s))$ .) Note that  $\tau$  is an increasing function, and it is

weakly concave in  $s$  if and only if  $\phi''(x)/\phi'(x) \equiv \tau'(\phi(x))$  weakly decreases with  $x$ .<sup>30</sup> In sum, we wish to maximize

$$\int_0^{u_{\max}} [V(r(u), u) + \alpha u][1 - G(r(u), u)]f(u)\tau(s(u)) du \quad (23)$$

subject to the endpoint condition  $s(u_{\max}) = 1$  and equation of motion (22). We proceed in three stages: (i) we show that an optimal solution exists; (ii) we derive necessary conditions for the optimal policy, and (iii) subject to a regularity condition we show that a policy satisfying the necessary conditions is a globally optimal policy.

First, that an solution to problem (23) exists can be deduced from the Filippov-Cesari Theorem (for instance, see Seierstad and Sydsaeter, 1987, chapter 2, Theorem 8). The only non-trivial requirement for this theorem to be invoked is that the set

$$N(s, u) = \{([V(r, u) + \alpha u][1 - G(r, u)]f(u)\tau(s) - \gamma, [1 - G(r, u)]f(u)\tau(s)) : \gamma \geq 0, v_{\min} \leq r \leq v_{\max}\}$$

be convex for each  $s$  and  $u$ . Write  $\eta(p, u) \equiv \int_{r(p, u)}^{v_{\max}} vg(v, u)dv$ , where  $r(p, u)$  is defined implicitly by  $G(r(p, u), u) \equiv 1 - p$ . Thus  $r(p, u)$  is the threshold such that a proportion  $p$  of projects lie above  $r(p, u)$  for given  $u$ , and  $\eta(p, u)$  is the integral of  $v$  above this threshold. Therefore,  $pV(r(p, u), u) = \eta(p, u)$ . Note that  $\eta$  is concave in  $p$ , and that the above set  $N$  is equal to

$$N(s, u) = \{([\eta(p, u) + \alpha up]f(u)\tau(s) - \gamma, pf(u)\tau(s)) : \gamma \geq 0, 0 \leq p \leq 1\},$$

which is convex since  $\eta(p, u) + \alpha up$  is a concave function of  $p$ . Therefore, an optimal strategy exists.<sup>31</sup>

Second, we describe the necessary conditions which must be satisfied by the optimal policy. *Pontryagin's Maximum Principle* (see Seierstad and Sydsaeter, 1987, chapter 2, Theorem 1) states that if a piecewise-continuous control variable  $r(\cdot)$  solves problem (23) then there exists a continuous and piecewise-differentiable function  $\lambda(\cdot)$  such that  $\lambda(0) = 0$  and for all  $0 \leq u \leq u_{\max}$ :

$$r(u) \text{ maximizes } (V(r, u) + \alpha u - \lambda(u))(1 - G(r, u)) \text{ for } v_{\min} \leq r \leq v_{\max} \quad (24)$$

<sup>30</sup>For instance, in the Poisson case  $\tau(s) = \mu s$ , and if there are two projects for sure then  $\tau(s) = 2\sqrt{s}$ . In general,  $\tau(1)$  is equal to the expected number of projects.

<sup>31</sup>Strictly speaking, the Filippov-Cesari Theorem shows the existence of a optimal *measurable* control  $r(u)$  rather than a piecewise-continuous control. However, in practice this is not an important limitation. (See Seierstad and Sydsaeter, 1987, chapter 2, footnote 9.)

and except at points where  $r$  is discontinuous

$$\lambda'(s) = (V(r(u), u) + \alpha u - \lambda(u))(1 - G(r(u), u))f(u)\tau'(s) . \quad (25)$$

Note that (24) implies

$$r(u) + \alpha u - \lambda(u) = 0 , \quad (26)$$

and so  $\lambda(u)$  represents the gap between the optimal rule  $r(u)$  and the naive rule  $r_{naive}(u) = -\alpha u$ . Since  $\lambda$  is continuous, it follows that  $r$  is itself continuous. Moreover, since  $r(\cdot)$  is continuous it follows from the Maximum Principle that  $\lambda(\cdot)$  is everywhere differentiable, in which case (26) implies that  $r(\cdot)$  is itself everywhere differentiable. Since  $\lambda(0) = 0$  it follows that  $r(0) = 0$ . Combining (25) and (26) yields

$$r'(u) + \alpha = (V(r(u), u) - r(u))(1 - G(r(u), u))f(u)\tau'(s(u)) ,$$

which is equation (7) in the text.

Finally, we discuss when a policy satisfying these necessary conditions is a global optimum. The *Arrow sufficiency theorem* (see Seierstad and Sydsaeter, 1987, chapter 2, Theorem 5) shows that the necessary conditions pick out a global optimum if

$$[V(r(u), u) - r(u)][1 - G(r(u), u)]f(u)\tau(s)$$

is concave in  $s$  for all  $u$ . However, since  $[V(r(u), u) - r(u)][1 - G(r(u), u)]f(u)$  is positive, the result follows if  $\tau$  is concave in  $s$ . This is so if and only if (8) holds.

**Proof of Proposition 2:** Condition (7) implies that at  $u = 0$  and any other  $u$  such that  $r_L(u) = r_H(u)$  we have

$$\frac{r'_L(u) + \alpha_L}{r'_H(u) + \alpha_H} = \frac{\zeta(x_L(u))}{\zeta(x_H(u))} . \quad (27)$$

If  $x_L(0) < x_H(0)$ , then by assumption (8)  $\zeta(x_L(0)) \geq \zeta(x_H(0))$ , and so (27) implies that  $r'_L(0) > r'_H(0)$ . In particular,  $r_L(u) > r_H(u)$  for small  $u > 0$ . If  $x_L(0) < x_H(0)$  then  $r_L(\cdot)$  must cross  $r_H(\cdot)$  at some point. (If  $r_L$  were uniformly above  $r_H$  then clearly the fraction of prohibited projects with  $\alpha_L$  would be greater than with  $\alpha_H$ .) Let  $u^*$  be the first point above zero where the curves cross. In particular, we have  $r'_L(u^*) \leq r'_H(u^*)$ . In addition, we must have  $x_H(u^*) > x_L(u^*)$  since  $x_H(0) > x_L(0)$  and  $r_H(u) \leq r_L(u)$  for  $u \leq u^*$ . But then (27) implies that

$$1 > \frac{r'_L(u^*) + \alpha_L}{r'_H(u^*) + \alpha_H} = \frac{\zeta(x_L(u^*))}{\zeta(x_H(u^*))} \geq 1 ,$$

a contradiction. We deduce that the curves can never cross, and so our initial assumption that  $x_L(0) < x_H(0)$  cannot hold.

**Proof of Proposition 3:** First note that if  $q_i^L \equiv q_i^H$  so that the two distributions coincide then the result clearly holds. So from now on suppose that  $q_i^L \neq q_i^H$  sometimes. Let  $\zeta_i(x) = \phi_i''(x)/\phi_i'(x)$ . We first show that MLRP implies that  $\phi_H'/\phi_L'$  strictly increases with  $x$ , i.e.,  $\zeta_H(\cdot) > \zeta_L(\cdot)$ . The derivative of  $\phi_H'/\phi_L'$  has the sign

$$\left( \sum_{n=2}^{\infty} n(n-1)q_n^H x^{n-2} \right) \left( \sum_{n=1}^{\infty} nq_n^L x^{n-1} \right) - \left( \sum_{n=2}^{\infty} n(n-1)q_n^L x^{n-2} \right) \left( \sum_{n=1}^{\infty} nq_n^H x^{n-1} \right).$$

We claim that the coefficient on each power  $x^N$ , for  $N \geq 0$ , in the above is non-negative. Defining  $a_k \equiv (k+2)q_{k+2}^H$  and  $b_k \equiv (k+2)q_{k+2}^L$ , the coefficient on  $x^N$  can be written as

$$\begin{aligned} \sum_{k=0}^N (k+1)(a_k b_{N-1-k} - a_{N-1-k} b_k) &= (N+1)(a_N b_{-1} - a_{-1} b_N) + \sum_{k=0}^{N-1} (k+1)(a_k b_{N-1-k} - a_{N-1-k} b_k) \\ &= (N+1)(a_N b_{-1} - a_{-1} b_N) + \sum_{k=0}^M [(N-k) - (k+1)](a_{N-1-k} b_k - a_k b_{N-1-k}), \end{aligned} \quad (28)$$

where  $M$  is the largest integer no greater than  $\frac{N-1}{2}$ . The final expression pairs together terms in  $(N-1-k)$  with terms in  $k$ . Since  $\frac{a_k}{b_k}$  is increasing in  $k$  by MLRP,  $\frac{a_{N-1-k}}{b_{N-1-k}} \geq \frac{a_k}{b_k}$  for all  $k \leq M \leq \frac{N-1}{2}$ . So every term in (28) is non-negative. However,  $q_i^L \neq q_i^H$  sometimes, the coefficient on at least some  $x^N$  must be strictly positive. It follows that  $\phi_H'/\phi_L'$  strictly increases with  $x$ , and hence that  $\zeta_H(\cdot) > \zeta_L(\cdot)$ .

If  $x_L(0) > x_H(0)$ , then we have

$$\frac{r'_L(0) + \alpha}{r'_H(0) + \alpha} = \frac{\zeta_L(x_L(0))}{\zeta_H(x_H(0))} \leq \frac{\zeta_L(x_H(0))}{\zeta_H(x_H(0))} < 1.$$

Here, the equality follows from (7), the first inequality follows since we assume that (8) holds for  $\phi_L$ , and the final inequality follows from  $\zeta_H > \zeta_L$ . We deduce that  $r'_L(0) < r'_H(0)$ . The rest of the proof follows the same lines (with  $L$  and  $H$  permuted) as that for Proposition 2.

**Proof of Proposition 4:** Consider first the impact of increasing  $\mu$ , and let  $\mu_L$  and  $\mu_H > \mu_L$  be two values for  $\mu$ . Let  $r_L(\cdot)$  and  $r_H(\cdot)$  be the corresponding optimal

threshold rules. From (13) it follows that at  $u = 0$  and any other  $u$  such that  $r_L(u) = r_H(u)$  we have

$$\frac{r'_L(u) + \alpha}{r'_H(u) + \alpha} = \frac{\mu_L}{\mu_H} < 1,$$

so  $r'_L(u) < r'_H(u)$  at all such  $u$ . So  $r_H$  can never cross  $r_L$  from above. We deduce that  $r_H(u) > r_L(u)$  for all  $u > 0$ . The argument for the impact of  $\alpha$  on  $r(\cdot)$  is similar.

**Proof of Proposition 5:** Let  $r(\cdot)$  be the candidate optimal threshold rule, and consider the impact on the principal's payoff in (19) of a small variation  $r(\cdot) + t\eta(\cdot)$  where  $\eta(\cdot)$  is an arbitrary piecewise-continuous function. Writing the principal's payoff (19) in terms of  $t$ , denoted  $W(t)$ , yields

$$W(t) = \alpha \int_0^{u_{\max}} u[1 - G(r(u) + t\eta(u), u)]f(u) du + \sigma' \left( \int_0^{u_{\max}} u[1 - G(r(u) + t\eta(u), u)]f(u) du \right) \left( \int_0^{u_{\max}} \left( \int_{r(u)+t\eta(u)}^{v_{\max}} vg(v, u)dv \right) f(u) du \right),$$

and so

$$W'(0) = -(B\sigma''(A) + \alpha) \left( \int_0^{u_{\max}} \eta(u)ug(r(u), u)]f(u) du \right) - \sigma'(A) \left( \int_0^{u_{\max}} \eta(u)r(u)g(r(u), u)]f(u) du \right).$$

Since  $W'(0)$  must equal zero for all  $\eta(\cdot)$ , it follows that  $r(\cdot)$  must satisfy (20).

## References

- AGHION, P., AND J. TIROLE (1997): "Formal and Real Authority in Organizations," *Journal of Political Economy*, 105(1), 1–29.
- ALONSO, R., AND N. MATOUSCHEK (2007): "Relational Delegation," *RAND Journal of Economics*, 38(4), 1070–1089.
- (2008): "Optimal Delegation," *Review of Economic Studies*, 75(1), 259–294.
- AMADOR, M., I. WERNING, AND G.-M. ANGELETOS (2006): "Commitment vs. Flexibility," *Econometrica*, 74(2), 365–396.
- BAKER, G., R. GIBBONS, AND K. MURPHY (1999): "Informal Authority in Organizations," *Journal of Law, Economics and Organization*, 15(1), 56–73.

- BERKOVITCH, E., AND R. ISRAEL (2004): “Why the NPV Criterion does not Maximize NPV,” *Review of Financial Studies*, 17(1), 239–255.
- FARRELL, J., AND M. KATZ (2006): “The Economics of Welfare Standards in Antitrust,” *Competition Policy International*, 2(2), 3–28.
- FRIDOLFSSON, S.-O. (2007): “A Consumer Surplus Defense in Merger Control,” in *The Political Economy of Antitrust*, ed. by V. Ghosal, and J. Stennek, pp. 287–302. Elsevier, Amsterdam.
- GREEN, J., AND J.-J. LAFFONT (1986): “Partially Verifiable Information and Mechanism Design,” *Review of Economic Studies*, 53(1), 447–456.
- HOLMSTROM, B. (1984): “On the Theory of Delegation,” in *Bayesian Models in Economic Theory*, ed. by M. Boyer, and R. Kihlstrom, pp. 115–141. Elsevier, Amsterdam.
- HOLMSTROM, B., AND P. MILGROM (1991): “Multi-Task Principal-Agent Analysis: Incentive Contracts, Asset Ownership and Job Design,” *Journal of Law, Economics and Organization*, 7(1), 24–52.
- LYONS, B. (2002): “Could Politicians Be More Right Than Economists? A Theory of Merger Standards,” mimeo, University of East Anglia.
- MARTIMORT, D., AND A. SEMENOV (2006): “Continuity in Mechanism Design without Transfers,” *Economics Letters*, 93(2), 182–189.
- MELUMAD, N., AND T. SHIBANO (1991): “Communication in Settings with no Transfers,” *RAND Journal of Economics*, 22(2), 173–198.
- SEIERSTAD, A., AND K. SYDSÆTER (1987): *Optimal Control Theory with Economic Applications*. North Holland, Amsterdam.
- SHIN, H. S. (2003): “Disclosures and Asset Returns,” *Econometrica*, 71(1), 105–133.
- SZALAY, D. (2005): “The Economics of Clear Advice and Extreme Options,” *Review of Economic Studies*, 72(4), 1173–1198.