

The Miracle as a Randomization Device

A Lesson from Richard Wagner's Romantic Opera *Tannhäuser und der Sängerkrieg auf Wartburg**

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Abstract

In this paper we provide textual evidence on the sophistication of medieval deterrence strategies. Drawing on one of the great opera librettos based on medieval sources, Wagner's *Tannhäuser*, we shall illustrate the use of optimal randomization strategies that can be derived by applying notions of dominance or trembling-hand perfection. Particular attention is paid to the employed randomization device.

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A very brief foreword.

Game theory provides extremely powerful tools to study social conflict and its applications have led to many straightforward solutions to apparently tricky problems. But can these solutions, provided by a theory that has basically been developed in the second half of the last century, be expected to have informed decisions before then? While there was clearly gravity before Newton's

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publication of the *Principia*, the issue of how the formulation of a theory affects the behavior of the objects it describes is less clear in the social sciences.¹

In this paper we provide textual evidence on the strategic savviness of the medieval church as portrayed in one of the great opera librettos based on medieval sources, Wagner's *Tannhäuser*.² We shall show that the church employed an optimal randomization strategy based on arguments of dominance or trembling-hand perfection. Particular attention is paid to the employed randomization device.

Introduction.

Crimes are committed because they promise an immediate benefit. The law and law enforcement try to countervail the criminal's incentives by threatening with deferred punishment and it is up to the potential criminal—to each one of us—to weigh the immediate benefits and the deferred costs when deciding about whether or not to commit a crime. A crucial role in this decision is assumed by the probability of apprehension.³ This is different for the sinner.

The (Christian) sinner who believes in an afterlife knows that a punishment is waiting for him for sure. The sinner will have to endure eternal sufferings in hell and for the range of usually observed discount rates this should provide rather strong incentives for behaving according to the rules set out by the (medieval Christian) church. However, there is one way out. The sinner can repent and atone and, if met by a forgiving priest, can be granted absolution.

Obviously, the enforcement agency—be it the state or the church—wants to deter aberrant behaviour but the deterrence mechanisms are subtly different for the two. While the state's representatives have to take action to punish, the church's have to take action to forgive. This is the simple consequence of the difference between an ignorant state (that does not observe the crime and has to exert effort to prosecute) and an all-knowing god (who does not overlook even the slightest misdemeanor).⁴ The defaults are exactly opposite. If nothing happens, the criminal goes

¹There is a small body of game theory literature with similar undertakings (some tackling fiction, some early legal codes); see Aumann and Maschler's (1985) and Aumann's (2003) studies of the Talmud; Brams' (1983, 1994, 2003) studies of the Hebrew Bible and other literary sources; Fudenberg and Levine's (2005) interpretation of the code of Hammurabi; and Mehlmann's (1989, 2000) game theoretic studies of Goethe's *Faust* and other stories.

²The opera's full title is *Tannhäuser und der Sängerkrieg auf Wartburg*. It was premiered in Dresden in 1845.

³For the large law & economics literature on optimal deterrence see, for example, Polinsky and Shavell (2000).

⁴Brams (1983) argues that superior beings, like the God of the Old Testament, sometimes appear to make mistakes and, thus, appear not to be omniscient. While we could make all the main points in our paper if we relaxed the assumption of omniscience, the exposition is much easier if we stick to the traditional idea of an all-knowing god.

free, the sinner is punished. Hence, deterrence should be much easier to achieve for the church (at least when facing a believer) and one wonders why the church would establish an institution whose purpose it is to weaken the default's deterrence? An institution that offers a way out to those who have done the very things the institution needs to discourage?

The answer is easily obtained and mirrors prominent arguments against the severest punishment offered by the state, the death penalty (which is the materialist's equivalent to eternal condemnation). Sinners and criminals alike who know that they will suffer the worst imaginable punishment anyway have no incentive to return to a path of decent, good behaviour once the sin or crime has been committed. They are condemned already and, hence, will seek any action that provides further immediate (net) benefits to them. There is nothing more dangerous than a first-time murderer who knows that he will face the gallows regardless of what he does next. And while this may be particularly bad news for police officers trying to apprehend the criminal, a condemned sinner is obviously bad news for everyone who surrounds him.⁵ Hence, both state and church also want to provide incentives to those who have fallen from grace to behave well again. This implies that there are two quite different constraints that will define optimal punishment technologies. Ex ante, an optimal punishment has to be severe enough to deter the crime or sin. Ex post, there must be a "way out" for those who "cooperate with the authorities."⁶ In modern game theoretic terms the second constraint can be derived from dominance arguments or the application of trembling-hand perfection: Even with perfect deterrence, bad things can happen, so you want to be prepared.

While the state may have many variables to play with to meet both constraints (simply because both, prosecution and punishment technologies, can be adjusted), we shall argue below that the church has only one option to balance the two goals and that option involves randomization. If absolution is always granted there is no deterrence. And if it is never granted there is no incentive for the sinner to repent. Thus, the choice whether or not to grant absolution must be random.⁷

Equipped with modern game theory, both, the application of trembling-hand perfection as well as the derivation of an optimal mixed strategy may appear straightforward. But can the

⁵Similar perverse incentives are created by debt overhang, see, for example, Krugman (1989).

⁶A worldly institution that offers rebates to "repentant" criminals is that of plea bargaining (see, for example, Reinganum 1988).

⁷Notice that the church cannot offer a "reduced" punishment for repentant sinners. It has only one "big" punishment at its disposal—hell—and this necessitates randomization. Dante's *Divina Commedia*, of course, knows different circles of hell, but for each sin, there is exactly one pre-specified circle and again there are no rebates. A couple of hundred years after Tannhäuser's death the Catholic church found, of course, another way to fine tune its deterrence mechanisms—by introducing purgatory (first defined in the Council of Trent, 1545-1563).

medieval church be expected to have followed such reasoning? In what follows we shall provide textual evidence from one of the great operas based on medieval sources, Wagner's *Tannhäuser*, to show that it did. Moreover, we shall document how randomization was achieved, namely by invoking the notion of a miracle.

The story and the model.

After being disenfranchised from his friends at the Wartburg Tannhäuser, a medieval Minnesinger, has chosen to visit the Venusberg, a place governed by the eponymous chieftain of sexual pleasure—a big no-no according to Christian beliefs. Nevertheless Tannhäuser has a jolly good time but, after a while, gets bored and decides that he has to return. Soon after being back in the green fields he missed so much he happens to meet his old friends from the Wartburg who, not knowing of his great sin, invite him back to the Wartburg. Initially hesitating, Tannhäuser decides to join them again once his friends remind him of his former love, the immaculate Elisabeth. He is greeted enthusiastically (not least by Elisabeth herself who has missed him quite awfully), and a song contest ensues that is to decide who will get to marry Elisabeth. For Tannhäuser, who is better equipped than all his competitors to win the contest, this is decision time. He can aggravate his sins by playing the part or he can confess.⁸

Given that we know that he is a believer, the implications of not confessing are obvious and dismal: The unrepentant sinner will eternally suffer in hell. But what are the consequences of confessions? The story has him walking to Rome to confess to the Pope, hoping for absolution.⁹ But before telling the end of the story let us try to model the problem. Employing the most basic game theoretic structure we can model the game between Tannhäuser and the Pope like this:

Stage 1 Tannhäuser decides about whether to sin or not. Sinning—visiting the Venusberg—promises substantial immediate gratification.¹⁰

⁸In a companion paper (Harmgart, Huck, and Müller 2006), more hermeneutic in its approach, we deal specifically with the song contest and the dilemma it imposes on Tannhäuser. For our purposes here though, we can abstract from this. Crucial is only that he has to take the decision, sooner or later, whether to confess or not.

⁹Tannhäuser's sin is exacerbated, as some authors argue, by having his life pledged earlier to Maria, the mother Jesus (see Moser 1977 or Borchmeyer 2004). Thus, the sin Tannhäuser committed in the Venusberg is a violation of his vow of celibacy. This is why, according to the Roman Catechism, the absolution of Tannhäuser can only be granted by the Pope.

¹⁰The music as well as Wagner's stage directions let very little doubt about how pleasurable a stay in the Venusberg is.

Stage 2 (that is only reached if Tannhäuser has sinned at Stage 1): Tannhäuser has to decide whether to confess or not. Deciding not to confess will bring him on a straight path to hell.

Stage 3 The church (the Pope in Tannhäuser's case) decides about whether to grant absolution or not.¹¹

On the basis of what we have discussed earlier, it appears obvious that once Stage 3 is reached the church should always prefer granting absolution to not to—simply because this ensures that the sinner (Tannhäuser in this case) has an incentive not to continue with his aberrant lifestyle. But this means immediately that there is a unique subgame perfect equilibrium outcome. Anticipating that he will be granted absolution Tannhäuser, the potential sinner, will decide to go to the Venusberg for a while and then repent (rationally expecting absolution once the deed is done). This gives him both, the immediate pleasure and a stab at eternal life in heaven. Consequently, the church has to *commit* itself in order to be able to achieve anything in this game. And given the church's rigid structure and its powers it seems reasonable to assume that the church can indeed commit to an absolution strategy for Stage 3. Essentially, this means that we add a Stage 0 to the game and drop Stage 3.

Stage 0 The church commits itself to an absolution strategy, i.e., it chooses a probability $p \in [0, 1]$ with which it grants absolution to a sinner who confesses in Stage 2.¹²

So, let us think about Tannhäuser's payoffs in this game and let us start by normalizing his payoff for not sinning to 0. If he sins (which provides him with some immediate pleasure) there are three possible outcomes for him:

1. He does not sin, a slightly boring but safe outcome that gives him a payoff that we normalize to zero.
2. He decides to sin, gains the immediate pleasure in the Venusberg, repents and is granted absolution. This is clearly better than not sinning at all and hence gives a strictly positive payoff that we shall denote by $b (> 0)$ as in benefit.
3. He gains the immediate pleasure, decides not to repent and to suffer in hell; a bad outcome that we shall normalize to -1 .

¹¹See also footnote 7.

¹²Notice that any such p is a *pure* strategy, i.e., the choice of a particular p means that the church will randomize with that probability *for sure*.

4. He gains the immediate pleasure, repents, but is not granted absolution; an outcome even worse than the second because he has to bear the costs of atonement without getting any benefit. We shall denote the payoff in this case by $-1 - c$ with $c > 0$ denoting the effort costs of atonement.

Assuming that eternal pains in hell are comparatively large we take it for granted that both b and c are comparatively small. The church can now analyse Tannhäuser's decision problem and then decide on an optimal absolution strategy. From our previous discussion it is clear that, first of all, the church wants to induce incentives for Tannhäuser not to sin. This obviously requires that absolution is not granted too easily and could, in fact, be achieved by never granting absolution, i.e., by basically abolishing Stage 3. More generally, however, it simply imposes a constraint on the *probability* with which absolution is granted. Denoting this probability by p we can state the first constraint on the church's optimal strategy as

$$pb + (1 - p)(-1 - c) < 0 \tag{1}$$

which can be rewritten as

$$p < \frac{1 + c}{1 + b + c}. \tag{1'}$$

As long as this constraint is met, Tannhäuser's expected utility from sinning is strictly negative and he will decide not to go the Venusberg—the first best outcome for the church (whose payoffs we need not model explicitly as only their ordinal structure matters for our main point). Intuitively, the constraint on p gets tougher the more pleasurable the sin (the higher b) and the smaller the costs of atonement (the smaller c). The default, $p = 0$, always meets the constraint and would be a good solution if the church could trust on Tannhäuser not making any mistakes. However, with the slightest “trembles”, i.e., with the slightest risk that, for whatever reason, Tannhäuser sins nevertheless, the church wants him to repent. This imposes a second constraint:

$$pb + (1 - p)(-1 - c) > -1 \tag{2}$$

which we can rewrite as

$$p > \frac{c}{1 + b + c}. \tag{2'}$$

In words, the probability of absolution has to be big enough to make confession worthwhile.

Taking the two constraints together, we get

$$\frac{c}{1 + b + c} < p < \frac{1 + c}{1 + b + c}. \tag{3}$$

Thus, there is an entire range of strategies fulfilling the two constraints, all of which involve *randomization*.¹³ Assuming that b and c are fairly small, the range is rather large. For a derivation of the *optimal* p we would need further assumptions. Realistically, the probability with which the potential sinner sins even if good deterrence is in place, i.e., if 1' holds, may depend on p and we could capture this, for example, by introducing a logistic choice function stemming from a random utility model. We would then also have to be explicit about the church's cardinal payoff structure. All of that would be straightforward but would not add much to the main point—that whatever the optimal (equilibrium) p it has to be in the interior. Heuristically, we might guess that if in doubt about which probability to pick, the church might feel more comfortable to make sure that its first-order target (to deter Tannhäuser from sinning) is achieved. Hence, it might wish to choose a rather small, albeit positive p .

While we were able to derive the optimality of randomization very easily, one may wonder whether the medieval church or medieval writers who were thinking about the church could have employed similar reasoning, reaching similar conclusions half a millennium before the advent of game theory. The lesson from Wagner's opera and its sources¹⁴ is that they obviously did—achieving randomization in a surprisingly elegant manner. So, let us now tell the rest of Tannhäuser's story.

He walks to Rome, always seeking out the most stony paths and avoiding the shelter of the shadows, as he wants to make sure that the pope takes his atonement seriously.

“The manner in which the heaviest-laden pilgrim beside me / took his way appeared to me too easy. / When his foot trod the soft sward of the meadows, / I sought thorn and stone for my bare feet; / when at the spring he would allow his lips to taste refreshment, / I would imbibe the scorching glow of the sun;”

¹³To the best of our knowledge, Schelling was the first author to point out that optimal commitment strategies might involve randomization—in his marvelous 1960 book. Notice, however, that the reasons for randomizations that he discusses are entirely different from the one introduced here. In all examples Schelling gives, randomization serves to lower the expected costs of a threat for the party who poses the threat. Cost reduction was probably also the logic behind the idea of decimation—a punishment strategy used in the Roman army that involved randomization. Those selected for punishment were divided into groups of ten; each group cast lots, and the soldier on whom the lot fell was executed by his nine comrades.

¹⁴The sources Wagner was drawing on tell the same story such that we can trace the ideas we discuss here much further back. The original Tannhäuser legend was first told in 1515. A popular account of the story through which Wagner probably learned about the legend is Ludwig Tieck's story *Der getreue Eckart und der Tannhäuser* first published in 1799. See, for example, Moser (1977) for more details on the genesis of the Tannhäuser story and Borchmeyer (2004) for more details on the genesis of Wagner's opera.

Alas, it is to no avail. When the pope hears that Tannhäuser has been to the Venusberg he shows his most unforgiving side. As Tannhäuser reports:

“And he whom I so begged began: — / ‘If you have enjoyed such sinful delights / and enflamed your passions at the fires of hell, / if you have sojourned in the Venusberg, / then, now from henceforth, you are eternally damned! / As this staff in my hand / no longer bedecks itself in fresh green, / so from the burning brands of hell / deliverance can never blossom for you!’ ”

On the surface, this appears pretty much like $p = 0$. “[T]hen, now from henceforth, you are eternally damned!” This does sound like game over. But the Pope continues with his speech and careful reading does suggest there might still be a tiny bit of hope for Tannhäuser. The pope’s staff—essentially a piece of dead wood—has to blossom again. If it does, so the implicit ruling, Tannhäuser will be pardoned after all.¹⁵

Of course, the chances of this happening are slim. But, as believers know, miracles can and do happen, so there is no doubt that $p > 0$. And, of course, we might already anticipate how the story ends—after all, what would be the point of elaborating on this sophisticated scheme if it doesn’t come to effect? And so it does. Made possible through Elisabeth’s sacrifice (who dies in grief when she hears from the Pope’s verdict)—the miracle occurs. As the choir of the pilgrims reports:

“It came to pass in the holy hour of night, / the Lord manifested Himself in a miracle. / The barren staff in a priest’s hand / He decked with fresh green: / for the sinner in the fires of hell / redemption shall blossom thus afresh!”

And so Tannhäuser dies—and goes straight to heaven:

“The salvation of grace is the penitent’s reward, / now he attains the peace of the blessed!”

A payoff of $+b$ after all.

A caveat.

¹⁵Notice that such “staff miracles” were more frequently referred to in the medieval literature. In fact, there are even other examples where the church explicitly demanded a miracle before granting salvation. Moser (1977) tells two intriguing stories in one of which the required miracle involves a black sheep that has to become white for salvation to be granted. In a second story a new-born child has to demand to be baptised by the sinner.

While our analysis shows that the staff miracle demanded for Tannhäuser’s absolution *can be viewed* as a clever incentive device some readers may not be convinced and argue instead that the miracle is much rather some sort of operatic *deus ex machina* to bring the story to a thrilling conclusion. But that, of course, would essentially amount to saying that the story is inherently weak. As Horace so poignantly put it in his *Ars Poetica* (1972, verse 191): “There should be no god to intervene, unless the problem merits such a champion.” And, in fact, Wagner’s libretto is sometimes criticized as weak and inconsistent in the opera literature (see, for example, Borchmeyer 2004).

However, if we are willing to view the Tannhäuser narrative as an empirical phenomenon this argument would violate the core principle of applied economic theory—to search for an “as if” rational choice or equilibrium interpretation of the observed outcome. In other words, there are only two options. One can give up the search for a rational core in the story and come to the conclusion that the story is weak or one can maintain the “as if” and realize that it is internally fully consistent. Of course, we want to advocate the latter. In fact, we believe that such an approach of what one might call “rational choice hermeneutics” provides a generally promising avenue for analysing narratives.

Conclusion.

In his famous book, Schelling (1960) discussed various reasons for why agents might want to employ randomization when it comes to threats and promises. The reason we discuss here, to mitigate the effects of deterrence and offer the one who has fallen a chance to return on a path of doing good, is not among them. In fact, we were not able to find any formalized argument of the type exemplified here, an argument that, however, is well known in discussions about capital punishment. In any case, it is derived easily. It follows simply from invoking trembling-hand perfection (or requiring dominance). And, thanks to Schelling, there is also not much surprise or awe when it turns out that the optimal commitment strategy in the game we study here, the game between the sinner and the pope, involves randomization. The church wants to commit itself to granting absolution stochastically.

Half a millennium ago, reaching these insights might have been far trickier. But as we have learned from studying the example of Wagner’s opera *Tannhäuser* (and its medieval sources) the church or, at least medieval writers who were thinking about the church, did solve the problem optimally and did employ a very effective randomization device—the miracle.

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APPENDIX

This appendix provides the original German text of the translated parts cited in the text.

“Wie neben mir der schwerstbedrückte Pilger / die Straße waltt’, erschien mir allzuleicht: — / betrat sein Fuß den weichen Grund der Wiesen, / der nackten Sohle sucht’ ich Dorn und Stein; / ließ Labung er am Quell den Mund genießen, / sog ich der Sonne heißes Glühen ein;”

“The manner in which the heaviest-laden pilgrim beside me / took his way appeared to me too easy. / When his foot trod the soft sward of the meadows, / I sought thorn and stone for my bare feet; / when at the spring he would allow his lips to taste refreshment, / I would imbibe the scorching glow of the sun;”

“Und er, den so ich bat, hub an: — / ‘Hast du so böse Lust geteilt, / dich an der Hölle Glut entflammt, / hast du im Venusberg geweilt: / so bist nun ewig du verdammt! / Wie dieser Stab in meiner Hand / nie mehr sich schmückt mit frischem Grün, / kann aus der Hölle heißem Brand / Erlösung nimmer dir erblühen!’ ”

“And he whom I so begged began: — / ‘If you have enjoyed such sinful delights / and enflamed your passions at the fires of hell, / if you have sojourned in the Venusberg, / then, now from henceforth, you are eternally damned! / As this staff in my hand / no longer bedecks itself in fresh green, / so from the burning brands of hell / deliverance can never blossom for you!’ ”

“Dahin zog’s mich, wo ich der Wonn’ und Lust / so viel genoß an ihrer warmen Brust! — / Zu dir, Frau Venus, kehr’ ich wieder, / in deiner Zauber holde Nacht; / zu deinem Hof steig’ ich darnieder, / wo nun dein Reiz mir ewig lacht!”

“It drove me there where I had enjoyed so much delight / and pleasure on her warm breast! / To you, dame Venus, do I return, / into thy magic’s sweet night; / to your court do I descend, / where your alluring charm will smile upon me now for always!”

“Es tat in nächtlich heil’ger Stund’ / der Herr sich durch ein Wunder kund: / den dürren Stab in Priesters Hand / hat er geschmückt mit frischem Grün: / dem Sünder in der Hölle Brand / soll so Erlösung neu erblühen!”

“It came to pass in the holy hour of night, / the Lord manifested Himself in a miracle. / The barren staff in a priest’s hand / He decked with fresh green: / for the sinner in the fires of hell / redemption shall blossom thus afresh!”

“Der Gnade Heil ist dem Büßer beschieden, / er geht nun ein in der Seligen Frieden!”

“The salvation of grace is the penitent’s reward, / now he attains the peace of the blessed!”