The Collusive Drawbacks of Sequential Auctions [§]

Gian Luigi Albano[†] and Giancarlo Spagnolo[††]

January 2005

Abstract

Sequential first-price auctions for multiple objects are very common in procurement, electricity, tobacco, timber, and oil lease markets. In this paper we identify two ways in which a sequential format may facilitate collusion among bidders relative to a simultaneous one. The first effect relates to the cartel's ability to identify and punish defectors within the sequence, thus lowering the gains from a deviation with respect to a simultaneous format. The second effect concerns the cartel's ability to allocate the bidder with the highest incentive to deviate (the 'maverick') to the last object of the sequence, thus increasing the viability of the collusive agreement. We then analyze how the seller may counteract this two effects by limiting the amount of information disclosed to bidders across rounds, and find that partial disclosure policies have little impact on the sustainability of collusion.

Keywords: Auctions, Collusion, Disclosure Policies
J.E.L. classification numbers: D44, L40, C72

[§]We are grateful to Philippe Jehiel, Nicola Pavoni and Francesco Squintani for insightful remarks, and seminar participants at the University of Bologna, ECARES-ULB and at the 13th WZB Conference on "Collusion and Cartels" for useful comments prior to this draft. This work is part of the programme for the ESRC Research Centre for Economic Learning and Social Evolution (ELSE).

[†]Department of Economics and ELSE, University College London, Gower Street, London WC1E 6BT, U.K. E-mail:g.albano@ucl.ac.uk, http://www.ucl.ac.uk/~uctpgla

[††]Consip Research Unit, Stockholm School of Economics and CEPR.

1 Introduction

Sequential first price auctions of multiple objects are very common. They are very often used in electricity, timber, and oil lease markets. In the US Tobacco market, sellers recently changed from a sequential open format to a sequential sealed bid first price format because bidders were perceived to collude.[1] In public procurement, supply contracts for different but related goods (for example printers, laptops, desktops, monitors, servers) are typically awarded separately, not simultaneously, using first price sealed bid auctions. For multiproduct bidders active on several of these goods, the procurement process turns into sequential first price auctions of multiple objects. Government securities and mineral rights, on the other hand, are typically sold in simultaneous auctions. This paper identifies two ways in which a sequential format may facilitate collusion among bidders relative to a simultaneous one, and analyzes disclosure policies that may counteract these effects.

In many real world sequential auctions there is full disclosure after each object is awarded, so we begin our analysis under this assumption. The first, intuitive collusive drawback of sequential auctions we identify is linked to the ability of ring members to identify defections from collusive strategies and react faster to them, within the sequence. This limits the short run gains a bidders can obtain by undercutting its cartel, facilitating collusion relative to a simultaneous format. This effect is related to that discussed by Admati and Perry (1991) for joint projects and by Neher (1999) for stage financing.[2] It is stronger the larger the number of goods sequentially auctioned (the smaller the lots in which a given divisible good is fractioned before being auctioned) and in the limit eliminates the enforcement problem.

The second effect we identify is linked to the possible asymmetry betwen cartel members. A common perspective on the viability of cartels is that it is limited by "maverick firms" (e.g., Baker 2002). Analogously, in our auction the viability of a ring is limited by maverick bidders, those with more to gain from undecutting collusive strategies. In a sequential setting, where asymmetric members of a bidding ring share the objects, the ring can facilitate collusion by allocating the most aggressive, maverick, bidder the last object(s) in the sequence. This minimizes the maverick's incentive to defect and increases the viability of the ring.

Both these effects are relative to what Comte *et al.* (2002) named cartel members' "punishment concern", that is, how large are the short run gains from defecting from a ring, and we will set up the model to keep our focus on this dimension. In the discussion we will argue though, that the "punishment concern", when relevant, can actually reinforce the pro-collusive effect of a sequential format.

Given that sequential auctions are simpler to organize than simultaneous ones, which is

[1] We are grateful to Peter Cramton for suggesting all these examples.

[2] See also Smirnov and Wait (2004).

probably the main reason why they are so common, we look at partial disclosure policies that could hinder these collusive drawbacks. We find that, among all possible partial disclosure policies, disclosing only the selling price of each object does somewhat reduce the cartel's ability to detect and punish internal deviations, but that most other partial disclosure policies have little effect. If the risk of collusion is high, the only disclosure policy with a strong impact is no disclosure whatsoever, that is, keeping all information on bids and winners on each lot secret for all bidders (including winners) until the end of the sequential auction.

Section 2 discusses a simple example. Section 3 lays out the model and discusses the main properties of the collusive mechanism. In Section 1, we compare the sustainability of collusion under the simultaneous and the sequential formats, and we perform some comparative statics analysis. Section 5 analyzes the effect of different possible disclosure policies. Section 6 offers a discussion and Section 7 briefly concludes. All proofs that do not appear in the main text are relegated to an appendix.

2 A simple example

Consider two identical bidders who plan to split collusively two identical objects sold through either a simultaneous or a sequential auction with zero reserve price where at the end of each round all information is disclosed. Monetary transfers are not feasible, bidders share the same valuation, $\overline{v} > 0$, for each of the objects, and this is common knowledge between them. The two bidders discount future profits according to the same factor δ_1 and are long-run 'competitors' that meet in a infinitely repeated versions of this or other market interactions. Sustaining collusion in the future is feasible and delivers discounted net (additional) collusive markup π_1 to each bidder, independent of the current auction format (this would be the case, for example, if the collusive markup comes from an unrelated repeated market interaction). Suppose further that collusion in the auction is supported by the threat of reverting to competitive behavior forever in case a deviation is observed. If the auction is simultaneous, the maximum gains from undercutting the split-the-objects collusive agreement to bid zero each on a different object and not to bid on the others is obtaining one additional object at a price marginally above zero, so that collusion is supportable in equilibrium if $\overline{v} \leq \delta_1 \pi_1$. It is immediate to check that precisely the same condition applies if the auctions are sequential. Therefore, with 2 symmetric objects and symmetric bidders we get an *irrelevance result.*

Suppose now, instead, that the two bidders, 1 and 2, are asymmetric. For example, assume that they have different access to credit markets, so that their discount factor differ and $\delta_1 < \delta_2$, which implies $\pi_1 < \pi_2$ (because they are the same streams of future profits but 2 discounts them at a lower rate than 1). Since both bidders must comply for a collusive agreement to be sustainable, if a simultaneous auction is used, the relevant condition for collusion being sustainable is the more stringent between $\overline{v} \leq \delta_1\pi_1$ and $\overline{v} \leq \delta_2\pi_2$. The "maverick" bidder 1 is constraining collusion to auctions where $\overline{v} \leq \delta_1\pi_1$ only; if $\delta_1\pi_1 \leq \overline{v} \leq \delta_2\pi_2$ the ring is not sustainable and Bertrand competition for the objects drive their price to $\overline{v}$. If the sequential auction is used instead, bidders may agree that the first object sold should be won by bidder 2, and the second by bidder 1. Now bidder 2 can still undercut bidder 1 on the second object and gain $\overline{v}$, so that the constraint $\overline{v} \leq \delta_2\pi_2$ remains relevant; but bidder 1 can now only undercut bidder 2 on the first object sold, which induces bidder 2 to revert to competitive behavior on the second object sold. Thus the maverik's gains from deviation fall to zero, and his incentive compatibility condition $0 \leq \delta_1\pi_1$ is always satisfied. With a sequential auction collusion becomes sustainable even when $\delta_1\pi_1 \leq \overline{v} \leq \delta_2\pi_2$.

Let us now go back to the case of symmetric bidders, but suppose there are three colluding bidders and 3 objects for sale (or 2 bidders and 4 objects). With a simultaneous format and a split-the-objects collusive agreement the critical constraint becomes $2\overline{v} \leq \delta_1\pi_1$ ($3\overline{v} \leq \delta_1\pi_1$ with 2 bidders and 4 objects), as each colluding bidder can undercut the other two (or tree) stealing them the objects at (almost) the collusive price. With a sequential format, on the other hand, a bidder undercutting one of the other two (or three) induces other bidders to bid competitively on the remaining one (or two), so that only one object can be gained through a unilateral deviation and the relevant incentive constraint becomes $\overline{v} \leq \delta_1\pi_1$ (also with 2 bidders and 4 objects). Hence, even with symmetric bidders, with more objects a sequential format facilitates collusion.

Both these pro-collusive effects of the sequential format are due to bidders' monitoring ability across different rounds of the sequential auction which reduces short run gains from defecting from the ring. The seller might then try to counterbalance the procollusive features of the sequential auction by making the mechanism *less transparent*. Of course, if the seller withholds all information on the outcomes of the various rounds of the sequential format until the end of the sequence, the sequential format becomes strategically identical to a simultaneous one and again we get an irrelevance result. But withholding all information until the end of the sequence may not be feasible. Partial diclosure policies where some information on the outcomes of the various rounds is withheld and some is released may be more acceptable. Suppose the seller withholds all information on the outcomes of the various rounds of the sequential auctions *but* the identity of the winner of each lot. Would this partial reduction of transparency reduce the pro-collusive effect of the sequential auction? It is clear that in our example this is not the case: it is easy to verify that announcing only the indentity of the winner of each object in the sequential auction the seller does not reduces bidders' ability to sustain collusion relative to the case of full transparency. However, there may be other partial disclosure policies that may reduce the pro-collusive effects of the sequential format. In the next sections we will deal with all these issues in a model that focuses on asymmetries in bidders' valuations.

3 A model of collusion among n asymmetric bidders under complete information

In this section, we first set up a more general model of collusion in simultaneous and sequential first-price auction able to capture the arguments in the example, and then discuss our main assumptions.

3.1 Primitives

The set of assumptions we shall be using throughout the paper is as follows.
A1. There are $n \geq 3$ homogenous objects; n dominant, colluding bidders with the highest valuations; and a fringe of $m - n$ independent bidders with lower valuations.
A2. Bidder i's private value of any single object is defined as

$$v_i \equiv \overline{v} + \lambda_i \sigma > 0,$$

where $\overline{v}$ and σ are positive constants and $i \in \{1, \ldots, n\}$. Moreover, bidder i's utility is linear in the number of objects, that is, if bidder i's buys k objects at a price of p each, her (net) utility becomes $k(\overline{v} + \lambda_1 \sigma) - kp$.
A3. $\lambda_1 > \lambda_2 > \ldots > \lambda_n$.
A4. λ_i is bounded away from $\pm\infty$, $i = 1, \ldots, n$.
A5. The seller's reserve price for each object is never binding, that is, valuations of all m bidders are strictly greater than the reserve price.
A6. [**Full Disclosure in the Sequential Auction**] In the sequential auction, the seller reveals all submitted bids and all bidders' identities at the end of each round.
A7. Asymmetry between the two highest-value bidders is not too large: $\overline{v} + \lambda_1 \sigma > n(\lambda_1 - \lambda_2)\sigma$, $\forall \sigma > 0$.
A8. Prices are discrete, the smallest monetary unit is ν.

Intuitively, we define private values in terms of distances from the average, where the distance is measured by the product of the standard deviation and a coefficient, λ_i, that is specific to each bidder i. The higher σ the more scattered private values, thus the higher the asymmetry among bidders. When $\sigma = 0$, the environment is perfectly symmetric. Assumption **A7** captures the relative distance between the two highest-value bidders. More precisely, it implies that bidder 2's and bidder 1's values are not too far apart.[3] The assumption also guarantees that bidder 1 is

[3]Notice that assumption **A7** can be re-written as $\frac{(\overline{v}+\lambda_2\sigma)}{(\overline{v}+\lambda_1\sigma)} > \frac{n-1}{n}$.

willing to join a bidding ring that allocates one object to each of its members. Under such a collusive scheme, the bidder with the highest value gets a collusive profit equal to $\overline{v} + \lambda_1\sigma$.[4] Should bidder 1 behave competitively, she would get each object at a price equal[5] to the second-highest value, $\overline{v}+\lambda_2\sigma$, which yields a net payoff of $(\lambda_1-\lambda_2)\sigma$. Thus **A7** ensures that bidder 1 has a strictly positive incentive to join the grand coalition. It is easy to see that all other individual rationality constraints are automatically satisfied. Indeed, competive bidding would leave all bidders $j = 2, \ldots, n$ with zero profit, whereas the collusive scheme guarantees strictly positive profit equal to $\overline{v} + \lambda_j\sigma$.

The auction mechanisms we consider are the simultaneous and the sequential auctions. In the simultaneous format, bidders simultaneously submit a demand function for the objects. The seller then awards the objects after aggregating demand functions to maximize her revenue. In the sequential format, the seller puts one object for sale at a time. Bidders simultaneously submit sealed bids for one object. The seller awards that object to the highest bidder at the highest price and makes all bids and bidder's identities public according to assumption **A6**. The seller then proceeds to the sale of the following object until all objects are sold. In both auction mechanisms, ties are broken by any random device that assigns the same probability of winning to each bidder. Finally, we assume that bidders' valuations are common knowledge among themselves, but the seller need not know them.

3.2 The Collusive Mechanism

If bidders were to compete for n objects only (under either auction mechanism), bidder 1 would be awarded all objects at a price (approximately) equal to the second-highest value.[6] Any collusive agreement aiming at reducing the seller's surplus must rely on some form of coordination among bidders. To this end, we assume that bidders interact repeatedly after the auction stage.

A9. The n colluding bidders share the same intertemporal discount factor δ and interact after the auction in a symmetric infinitely repeated market interaction (Bertrand oligopoly, auction) where they sustain collusion with "grim trigger" strategies; in this collusive subgame, players' incentive compatibility conditions are satisfied as equalities.

A10. The collusive ring at the auction stage is also supported by grim trigger strategies, that is, by conditioning collusion in the following oligopolistic sub-game on the absence of defections from the collusive bids agreed for the auction. Bidders share equally the stakes form collusion tomorrow if and only if they adhere to a well-defined strategy at the auction stage today.

[4] We are assuming the collusive price to be zero. The assumption is more clearly stated and discussed in Section 3.2.

[5] Given the assumption of discrete prices, the selling price would equal to the second-highest value plus ν. Since ν is assumed to be very small, we will suppress it form all relevant expressions.

[6] At the (unique) Nash equilibrium in undominated strategies, the price of each object would be $(\overline{v} + \lambda_2\sigma) + \nu$.

A11. As customary in most models of collusion, we assume that side transfers among bidders are not feasible. We also assume that collusive allocations involving equal bids and randomization are excluded because they substantially increase the likelihood to be discovered and fined by an Antitrust Authority.

A12. To simplify and keep focus on cartel enforcement, we also assume that colluding bidders can perfectly forecast the highest bid submitted by the fringe, and we normalize the highest bid from the fringe to zero.

Assumptions **A9-A12** ensure that the collusive allocation among the n bidders takes the form of a classic 'split-award' scheme, the type of collusive agreement where the effect of allocating to the maverick the last object(s) auctioned in a sequential auction as discussed in the example is relevant (all other effects we highlight would emerge in any other collusive scheme; see the discussion at end of the paper).

The only sustainable collusive allocation then takes the following form: at the auction stage each colluding bidder submits a bid just marginally higher than the highest fringe bid on one object, and lower (fake) or no bids on other objects. Thus, each bidder in the ring is awarded exactly one object.

The repeated interaction described by **A9** provides the punishment phase necessary to enforce the ring. Define Π the expected value of collusion among bidders from tomorrow's market interaction. If the allocation is implemented, then each bidder gets an equal share of the collusive profit at the post-auction stage Π/n. If, instead, a defection takes place at the auction stage, we assume that the collusive profit at the post-auction stage shrinks to zero owing to cut-throat competition.

The seller need not be aware of the active cartel. The literature on collusion in auctions[7] has stressed the role of a secret reserve price as an effective way to fight a bidding ring. However, it has been pointed out that such a strategy requires a strong commitment power on the seller's side, that is, her willingness not to sell the object when the highest submitted bid is lower then the (secret) reserve price. In this paper, we pursue a different line of investigation. The reserve price is exogenously given, but the seller can vary the degree of transparency of the sequential mechanism as anti-collusive device. Varying the degree of transparency of a selling mechanism also requires some commitment power, since the seller has to withhold some pieces of information until all the objects have been sold. However, by using the latter strategy the seller never runs the risk of not selling the objects. This objective is often considered paramount in procurement auctions.

[7]See, for insatnce, McAfee and McMillan (1992)

4 Collusion with a fully transparent sequential auction

In this section, we compare the sustainability of collusion in the two auction formats under the assumption that the seller publicly announces the winner's identity and the selling price at the end of each round of the sequential auction (assumption **A6**). Moreover, a simple comparative statics analysis will show that higher asymmetry among bidders makes collusion easier to sustain in the sequential *relatively* to the simultaneous auction. We shall then consider the situation, frequently arising in procurement auctions, in which each good is fractioned in multiple lots.[8] When lots are sold sequentially, monitoring becomes more effective since the gains from defection are lower the higher the number of lots. At the limit, when the goods put for sale become perfectly divisible items, the defection strategy yields zero to each bidder which, in turn, implies that collusion in the sequential auction is sustainable for any discount factor.

The first result illustrates that the highest-value bidder's gains from defections can be minimized by allocating that bidder to the last object in the sequential auction.

Lemma 1 ***[The maverick last]*** *Suppose that bidders are asymmetric, that is, $\sigma > 0$. Then,*
(i) the optimal collusive agreement in the sequential auction allocates object n to bidder 1 (the "maverick");
(ii) the order with which all other bidders obtain their object is irrelevant.

Proof. *(i)* Bidder i's incentive compatibility constraint at the sequential auction writes

$$\Pi_{seq}^{D_i} \leq \Pi_{seq}^{C_i} + \delta \frac{\Pi}{n}, \tag{1}$$

where $\Pi_{seq}^{D_i}$ and $\Pi_{seq}^{C_i}$ are bidder i's payoffs from the optimal deviation and from adhering to the collusive agreement respectively. Suppose that the collusive agreement allocates bidder 1 to object n and bidder $j \neq 1$ to object $k \neq n$. Call this allocation the 'candidate allocation'. When bidder 1 gets object n, her most profitable deviation is to become active on object 1. Thus bidder 1 gets one object at the minimal collusive price and reaps the competitive profit on all other $(n-1)$ objects, that is, $\Pi_{seq}^{D_1} = (\overline{v} + \lambda_1 \sigma) + (n-1)(\lambda_1 - \lambda_2)\sigma$. If the collusive agreement allocates bidder 1 any other object k different from n, bidder 1's set of possible deviations becomes larger. Bidder 1 could choose between deviating on object 1 and deviating on object $k+1$. The candidate allocation minimizes the space of bidder 1's possible deviations.
(ii) Consider now bidder $j \neq 1$. Under any collusive allocation, $\Pi_{seq}^{D_j} = 2(\overline{v} + \lambda_j \sigma)$, $j \neq 1$. Indeed

[8] One example is the procurement contract to provide a service or supply a certain commmodity in a region that might be fractioned in, say, k contracts for k homogeneous geographical areas.

the most profitable deviation to bidder j is to become active on the object after the one assigned by the collusive agreement. Thus, each bidder $j \neq 1$ can get at most two objects by defecting from the collusive agreement. ■

Before presenting the main result of the this section, we need some additional pieces notation. The presence of asymmetry among bidders implies that each of them will have different incentives to defect from and to adhere to the collusive scheme. Thus each bidder will have a specific discount factor at which she will be willing to stick to the collusive agreement. We order the n relevant discount factors according to their stringency and define $\delta_f^{(i)}(\sigma)$ the $i - th$ more binding incentive incompatibility constraint in auction format f, where f could be either the simultaneous (sim) or the sequential (seq) auction. Thus collusion in the auction format $f \in \{sim, seq\}$ is sustainable if and only if $\delta(\sigma) \geq \delta_f(\sigma) \equiv \delta_f^{(1)}$.

We are now in a position to state

Proposition 1 *Assume A1-A12. Then*

(i) The sequential auction format makes collusion easier to sustain than the simultaneous format regardless of the degree of asymmetry among bidders, that is, $\delta_{sim} > \delta_{seq}, \forall \sigma \geq 0$*;*

(ii) When bidders are sufficiently symmetric, the relevant incentive constraint in the sequential auction is weaker than the slackest incentive constraint in the simultaneous format, that is, there exists a threshold degree of asymmetry among bidders σ^* *such that for all* $0 < \sigma < \sigma^*$, $\delta_{seq}(\sigma) < \delta_{sim}^{(n)}(\sigma)$.

Part *(i)* formalizes the basic difference between the simultaneous and the sequential auctions. Under the collusive device, each bidder is awarded one object at the lowest possible price which is zero. Thus the gains from collusion coincide under the two different auction formats. The gains from defection, however, are different. In the simultaneous auction, each bidder can get all objects by submitting a bid of ν on all objects. This format maximizes each bidder's gains from defection. The informational features of the sequential auction, instead, allow bidders to perfectly monitor across rounds the adherence to the collusive agreement. If a defection occurs at any round j, a competitive bidding unfolds, if $j < n$, and the additional collusive profits in the post-auction interaction, Π, will shrink to zero. Each bidder's gains from defection are strictly lower than in the simultaneous auction, and this makes collusion more easily sustainable in the sequential format for any given degree of asymmetry among bidders.

Part *(ii)* emphasizes that, when bidders are sufficiently alike, the relevant incentive constraint in the sequential auction satisfies *all* incentive constraints arising in the simultaneous format. When bidders' valuations are not far apart, the value of the deviation is mainly determined by the number of additional objects each defecting bidder can obtain with respect to the collusive scheme. If σ is small enough, bidder 2's additional payoff from a deviation in the sequential auction is close to $\overline{v}$, whereas bidder n's additional payoff from a deviation in the simultaneous auction is close to $(n-1)\overline{v}$. This explains why the relevant constraint in the sequential format becomes less binding the slackest constraint, that is, bidder n's, in the simultaneous auction.

The next result provides some comparative statics analysis on the absolute and relative sustainability of collusion in the two auction formats. The results are illustrated in figure 1.

Proposition 2 *When bidders become more asymmetric,*

(i) collusion becomes more difficult to sustain in both auction formats, that is, $\dfrac{d\,\delta_{sim}(\sigma)}{d\,\sigma} > 0$, *and* $\dfrac{d\,\delta_{seq}(\sigma)}{d\,\sigma} > 0$*;*

(ii) collusion becomes easier to sustain in the sequential format relatively to the simultaneous format, that is, $\Delta(\sigma) \equiv \delta_{sim}(\sigma) - \delta_{seq}(\sigma)$ *is strictly increasing in* σ*.*

Proof.
(i) It is immediate that $\dfrac{d\,\delta_{sim}(\sigma)}{d\,\sigma} = \dfrac{n(n-1)\lambda_1}{\Pi} > 0$ and $\dfrac{d\,\delta_{seq}(\sigma)}{d\,\sigma} = \dfrac{n\lambda_2}{\Pi} > 0$ due to **A1** and **A3**.
(ii) We have that

$$\frac{d\,\Delta(\sigma)}{d\,\sigma} = \frac{n[(n-1)\lambda_1 - \lambda_2]}{\Pi} > 0,$$

due again to **A1** and **A3**. ■

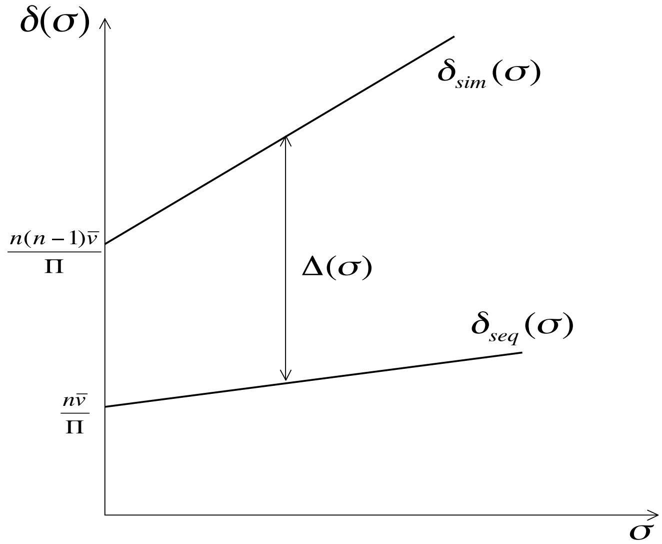

Figure 1: Comparative statics on the degree of asymmetry

4.1 Increasing the number of lots

If the objects auctioned off are divisible goods, a crucial aspect of the auction design concerns the number of lots in which each single object is fractioned. In offshore oil lease auctions, for instance, both the drainage and the wildcat tracts may have different sizes; in procurement auctions for the construction of new highways, the project can be fractioned in few and long lots rather than in many and short ones. Varying the number of lots does have a substantial impact on the sustainability of a collusive agreement. In general, the higher the number of lots, the easier is to sustain collusion. At the limit, when each object becomes perfectly divisible, cartel enforcement disappears in the sequential auction. In the simultaneous format, instead, the critical discount factor for collusion is always bounded away from zero.

Before stating these results formally, we amend the basic framework and assume that each of the n objects put for sale is fractioned in k homogenous lots. Thus bidder i's valuation of each lot is simply $\frac{\overline{v}+\lambda_i\sigma}{k}$. The auction formats remain unchanged, although the sequential auction requires nk round to be completed. The next result shows that, as k grows large, the sustainability of collusion is not affected in the simultaneous auction when bidders are perfectly symmetric, whereas it becomes easier to sustain when asymmetries are present.

Proposition 3 *Suppose the auction format is simultaneous. Then*

(i) if bidders are symmetric, collusion does not depend on k;
(ii) if bidders are asymmetric, the minimal discount factor at which collusion is sustainable weakly decreases with k and is bounded away from zero.

Part (*i*) is immediate. When bidders are symmetric, lots are allocated evenly among them, which implies that each bidder's payoff from adhering to collusive agreement does not depend on k. Moreover, the value of a deviation does not change with the number of lots (a defecting bidder optimally deviates on all lots) either, so all incentive compatibility constraints are independent of k. When bidders are asymmetric (part (*ii*)) and each object is fractioned in at least two lots, the collusive agreement can now allocate a higher number of lots to high value bidders. The split-the-market nature of the cartel becomes now more flexible. Given that each bidder must get (at least) one lot, the cartel can redistribute (at most) $(n-1)k$ lots according to the degree of asymmetry.

Next we consider the effect of increasing the number of lots in a sequential auction. Since a defection is immediately detected, the gain from a deviation coincides with the value of the lot for the defecting bidder. Thus the higher k the lower the gain from defection. When k grows arbitrarily large the problem of cartel enforcement disappears altogether.

Proposition 4 ("Folk Theorem" for the Sequential Auction) *As k grows arbitrarily large, the minimal discount factor at which collusion is sustainable in the sequential auction tends to zero. Hence, for any σ and any $\delta < 1$, there exists a finite k such that collusion is sustainable in the sequential auction.*

Before analyzing the effects of a limited information disclosure to bidders across rounds on the sustainability of collusion, we investigate two extensions of the current framework.

4.2 Two "mavericks"

The sustainability of collusion in the sequential auction crucially depends upon the asymmetry between the highest and the second-highest value bidders. Indeed the difference between the two determines the amount of non-collusive profit to the highest-value bidder (the maverick). We know from lemma 1 that bidder 1's optimal deviation from the most collusive allocation in the sequential auction yields $(\overline{v}+\lambda_1\sigma)+(n-1)(\lambda_1-\lambda_2)\sigma$, where the first term captures the profit on object 1, and the second term measures the amount of profit stemming from the resulting Bertrand competition on objects $2, \ldots, n$.

It would then be tempting to conclude that the presence of a second maverick, that is, of another bidder whose value for an object is $\overline{v} + \lambda_1\sigma$, would reduce the first maverick's incentive to deviate since any Bertrand competition would result in zero profit to *all* bidders. A closer inspection will however reveal that this intuition is not always true. To see this, amend the set of bidders so that the low-value bidder (bidder n) is replaced with a second, high-value bidder. Thus, at the collusive allocation, each member of the coalition is still awarded exactly one object. Notice that under this collusive allocation, each bidder's participation constraint is automatically satisfied. Since Bertrand competition yields zero profit to *all* bidders, assumption **A7** becomes superfluous.

In this modified framework, it is still true that the optimal collusive device requires one maverick, say 1, to be allocated object n. Then, the gains from a defection on object 1 to maverick 1 are reduced to $(\overline{v} + \lambda_1\sigma)$, since a defection on object 1 would trigger a competitive bidding on object 2 to n that results in zero profit to all bidders. Moreover, suppose that maverick 2 is allocated any object $i \in \{1, \ldots, n-1\}$. It is easy to see that maverick 2's optimal deviation is to defect on object $i+1$ which yields a payoff of $2(\overline{v} + \lambda_1\sigma)$. Thus the presence of a second maverick lowers maverick 1's gains from defection by $(n-1)(\lambda_1 - \lambda_2)\sigma$, and increases maverick 2's gains from defection by $(\overline{v} + \lambda_1\sigma)$, since, by cheating, the latter can get two objects. We can then summarize our finding in the following

Proposition 5 *Adding a second maverick is (i) procollusive if* $(\overline{v} + \lambda_1\sigma) < (n-1)(\lambda_1 - \lambda_2)\sigma$*; (ii) anticollusive if* $(\overline{v} + \lambda_1\sigma) > (n-1)(\lambda_1 - \lambda_2)\sigma$*.*

Corollary 1 *When bidders are symmetric enough, adding a second maverick always has an anticollusive effect.*

Proof. When $\sigma = 0$, the effect is always anticollusive since $\overline{v} > 0$. Given that λ_1 is assumed to be bounded away from $\pm\infty$, it must exists a threshold value σ' such that for all $\sigma < \sigma'$, we have that $(\overline{v} + \lambda_1\sigma) > (n-1)(\lambda_1 - \lambda_2)\sigma$. ■

4.3 Synergies and Package Bidding

Synergies in multi-unit auctions arise when bidders value a group of objects more than the sum the values of each individual object. Synergies are typically generated by economies of scale. In the US spectrum auctions, for instance, they arise from infrastructure cost savings whenever two licenses correspond to two neighboring regions. In order to capture this effect, we define bidder i's utility from owning y objects as $u_i(y=1) = \overline{v} + \lambda_i\sigma$, and $u_i(y \geq 2) = y(\overline{v} + \lambda_i\sigma)(1+\alpha)$, where $\alpha > 0$. The multiplicative nature of the synergies is meant to captures the fact that a bidder's cost savings are proportional to the number of object awarded to the same bidder. Let now $\delta^{\alpha}_{sim}(\sigma)$ and $\delta^{\alpha}_{seq}(\sigma)$ denote the relevant discount factor for collusion to be sustainable in the simultaneous and the sequential auctions respectively when synergies among objects are present. Moreover, let $\Delta^{\alpha}(\sigma) \equiv \delta^{\alpha}_{sim}(\sigma) - \delta^{\alpha}_{seq}(\sigma)$. The following result compares the sustainability of collusion in the two auction formats when positive synergies are present.

Proposition 6 *Suppose that positive (multiplicative) synergies exist among the objects put for sale. Then, for any given degree of asymmetry among bidders, (i) collusion becomes more difficult to sustain in both the simultaneous and the sequential auction, that is,* $\delta^{\alpha}_{sim}(\sigma) > \delta_{sim}(\sigma)$ *and* $\delta^{\alpha}_{seq}(\sigma) > \delta_{seq}(\sigma)$, $\forall\sigma$*; (ii) the relevant discount factors for collusion in the two formats vary exactly by the same amount, that is,* $\dfrac{d\,\Delta^{\alpha}(\cdot)}{d\,\alpha} = 0$*; (iii) allowing for package bidding does not modify the sustainability of collusion in the simultaneous auction.*

5 Disclosure policies

The rules of the sequential auction analyzed so far generate a simple pro-collusive effect. When the seller adopts a fully transparent sequential auction, bidders are able to perfectly monitor each other at each single round, thus the gains from defection are always lower than in a simultaneous auction. This section explores how the sustainability of collusion in the sequential auction varies with the amount of information disclosed by the seller.

If no information whatsoever is disclosed to bidders until the end of the sequential auction, of course the sustainability of collusion coincides in the sequential and the simultaneous formats. Our focus here is on partial disclosure policies, where the seller witholds only part of the information.

We shall consider two different scenarios. In the first scenario, that we name uniform disclosure policies, the seller may choose to limit the amount of information released to bidders at the end of each round, but *all* bidders receive exactly the same amount of information. In the second scenario, that we name discriminatory disclosure policies, bidders are treated asymmetrically. The seller may then release different amount of information to each bidder at the end of each round.

5.1 Uniform Disclosure Policies

The pieces of information that the seller can disclose at the end of each round of the sequential auctions are: the selling price, the winners' identities, the losing bids and the non-winners' identities. The main result of this section states that partial disclosure policies are ineffective in making collusion less sustainable in the sequential auction. If the sellers is constrained to adopt uniform disclosure policies, collusion can be reduced by withholding all information[9] until the last round, thus transforming the sequential format into a simultaneous one.
We can now state the following.

[9]This case of *minimal information disclosure* is somewhat related to Blume and Heidhues (2003). They consider an infinitely-repeated first-price auction in which after any stage each bidder is informed only about whether or not she has won the object. They show that when bidders are patient enough collusion can still be supported by a bid-rotation scheme.

Proposition 7 *Any disclosure policy that reveals the winner's or the non-winners' identities or the losing bids does not affect the sustainability of collusion in the sequential auction. Moreover if the seller releases the selling price only, and defecting with a bid equal to the collusive one exposes the defecting bidder to a high risk of being fined by the Antitrust Authority, then collusion does not becomes less sustainable than under a policy of full transparency.*

Proof. Suppose first that the seller discloses only the winner's identity at the end of each round. Any deviation would then result in the seller announcing a winner different from the collusive allocation. Thus any deviation is immediately detected. Notice that any other partial disclosure policy that includes the winner's identity would yield the same outcome.

Suppose now that the seller discloses only the non-winners' identities at the end of each round. Any deviation at round i would then result in the seller announcing a list of non-winners that includes also the bidder who should have been won object i according to the collusive agreement. Such a defection is immediately detected.

Third, consider the disclosure policy that only makes the losing bids public. If a deviation takes place, there will be a bid equal to the collusive price on the list of losing bids. The deviation is, again, immediately detected.

We are then left with the selling price. If bidders are only informed about the selling price, a defecting bidder can only deviate by submitting a bid different from the collusive one. This implies that that the deviation is immediately detected by all other bidders.

Under each of these four scenarios, the immediate detection of a deviation implies that the gains form defection coincide with those under a policy of full transparency. Thus the relevant discount factor for collusion to be sustainable under any uniform disclosure policy coincides with the one under a policy of full transparency. ■

Remark. If defecting with a bid equal to the collusive price on *all* objects, does not raise too much the risk of an antitrust investigation, the policy of revealing only the selling price may become effective in making collusion less sustainable in the sequential auction. Under this scenario, the optimal deviation strategy consists in submitting a bid identical to the collusive price on objects 1 to $n-1$, and a bid of ν on object n.[10] Thus the seller awards $n-1$ objects by a flip of a coin. This strategy yields bidder i a defection payoff of $\frac{n-1}{2}(\overline{v}+\lambda_i\sigma)+(\overline{v}+\lambda_i\sigma)$ which is greater (strictly for $n \geq 4$) than the defection payoff under a policy of full transparency. The incentive compatibility constraints for all bidders become more stringent, thus the relevant discount factor for collusion

[10]Consider bidder $i = 2, \ldots n$. By submitting a bid equal to the collusive price - that is, zero - on object $k, 1 \leq k \leq n-1$, and a bid of ν on object $k+1$, bidder i gets $\frac{k}{2}(\overline{v}+\lambda_i\sigma)$. This value is maximized at $k = n-1$. If bidder 1 adopts the same strategy then her pay-off becomes $\frac{k}{2}(\overline{v}+\lambda_1\sigma)+(\overline{v}+\lambda_1\sigma)+(n-k-1)(\lambda_1-\lambda_2)\sigma$, which is also maximized at $k = n-1$.

when the seller discloses only the selling price is higher (strictly for $n \geq 4$) than the one under full transparency. Although we do not explicitly model the behavior of an Antitrust Authority, we will keep the assumption that an antitrust investigation is promoted as soon as equal bids are submitted on any of the n objects. The expected fine is assumed to be high enough to make such bidding pattern unprofitable for colluding bidders.

5.2 Discriminatory Disclosure Policies

Discriminatory disclosure policies allow the seller to discriminate bidders according to the amount of information revealed at the end of each round. There exist, however, legal constraints in procurement auctions that force the procurement agency to put the winner at the end of each round in a privileged position. For instance, the Italian procurement agency (Consip) has to immediately inform the winner that she won an object. We will keep this assumption throughout, and ask how much information the seller has to release to the *non-winning* bidders in order to make collusion more difficult to sustain than under a policy of full transparency.

The relevant pieces of information disclosure coincide with those analyzed in the case of uniform disclosure policies: the selling price, the winner's identity and the losing bids (the non-winners' identities are automatically implied by the discriminatory nature of the disclosure policy).

Suppose first that at the end of each round the seller informs the winner that she has won the object, and discloses only the winner's identity to the non-winning bidders. This scenario corresponds to the case in which all bidders are *symmetrically* informed about the winner's identity at the end of each round, so collusion is as sustainable as under a policy of full transparency. Suppose next that the seller discloses only the losing bids to the non-winners. Should a defection take place, the set of losing bids would include one bid equal to the collusive price that should have been the winning bid. Thus a defection is immediately detected by all bidders, and again collusion is not affected by such a discriminatory disclosure policy.

We are then left to consider two relevant discriminatory disclosure policies. The first involves the seller disclosing only the selling price to the non-winners. Call this disclosure "PA" (price announcement), and define δ_{seq}^{P} the relevant discount factor for collusion. The second, extreme, discriminatory disclosure policy is the one under which the winner is informed that she has been won one object whereas all other bidders receive no information. Call "ND" (no disclosure) this disclosure policy, and define δ_{seq}^{N} the relevant discount factor for collusion. In order to state our final result we need the following assumption

A13. $\overline{v} + \lambda_2\sigma < (n-1)(\lambda_1 - \lambda_3)\sigma$, $\forall \sigma > 0$.

Taken together assumptions **A7** and **A13** can be rewritten as

$$(n-1)\underbrace{(\lambda_1-\lambda_3)\sigma}_{(v_1-v_3)} > \underbrace{(\overline{v}+\lambda_2\sigma)}_{v_2} > (n-1)\underbrace{(\lambda_1-\lambda_2)\sigma}_{(v_1-v_2)}$$

We can now state the following

Proposition 8 *Suppose that at the end of each round the seller informs the winner that she has won an object, and either provides no information to all other bidders ("ND" policy) or only discloses the selling price ("PA" policy). Moreover, suppose that defecting with a bid equal to the collusive one exposes the defecting bidder to a high risk of being fined by the Antitrust Authority. Then,*
i) the sustainability of collusion under both discriminatory disclosure policies coincides with the case of full disclosure when bidders are symmetric, that is, $\delta^N_{seq}(\sigma) = \delta^P_{seq}(\sigma) = \delta_{seq}(\sigma)$ *when* $\sigma = 0$*;*
ii) when bidders are asymmetric, regardless of whether colluding bidders can communicate, collusion is not affected by the "PA" policy, and becomes more difficult to sustain under the "ND" policy than in the case of full disclosure, that is, $\delta^N_{seq}(\sigma) > \delta_{seq}(\sigma)$*, and* $\delta^P_{seq}(\sigma) = \delta_{seq}(\sigma)\,\forall\sigma > 0$.

Informing the winner at the end of each round while keeping all other bidders in a state of ignorance implies that any deviation taking place at stage t is observed only by two bidders: the defecting bidder and the one who should have won the object according to the collusive scheme. The reversion to a 'competitive' bidding involves only two bidders at stage $t+1$, three bidders at stage $t+2$ etc. This imperfect contagion argument implies that the gains from defection are higher than under a fully transparent sequential auction. However, the adoption of the "ND" policy makes the computation of the *optimal* defections more difficult to be pinned down since they depend upon the collusive assignment of *all* bidders and not only the position of the maverick. The following example illustrates this last point.

Example 1. Assume that the seller adopts the "ND" policy. There are $n = 4$ bidders, bidders 3 and 4 have the same valuation for each object, that is, $\lambda_3 = \lambda_4$. Consider the following collusive allocation (**AL1**): object 4 is assigned to bidder 1, bidder $i = 2, 3, 4$ is assigned to object $i-1$.
If

$$3(\lambda_1-\lambda_2) > 2(\lambda_1-\lambda_3),$$

bidder 1 optimal deviation involves becoming active on object 1, which implies the following incentive compatibility constraint

$$\begin{aligned}(\overline{v}+\lambda_1\sigma) + 3(\lambda_1-\lambda_2)\sigma &\leq (\overline{v}+\lambda_1\sigma) + \delta\frac{\Pi}{n} \\ \delta &\geq \delta_1 \equiv \frac{n}{\Pi}\left[3(\lambda_1-\lambda_2)\sigma\right].\end{aligned}$$

Given **AL1**, bidder 2's IC constraint becomes

$$\begin{aligned} 2(\overline{v} + \lambda_2\sigma) + 2(\lambda_2 - \lambda_3)\sigma &\leq (\overline{v} + \lambda_2\sigma) + \delta\frac{\Pi}{n} \\ \delta &\geq \delta_2 \equiv \frac{n}{\Pi}\left[(\overline{v} + \lambda_2\sigma) + 2(\lambda_2 - \lambda_3)\sigma\right]. \end{aligned}$$

Collusion is thus sustainable if and only if $\delta \geq \delta_{seq}^N \equiv \max\{\delta_1, \delta_2\}$. Suppose that the following assumption holds
H1: $(\Delta_1 - \Delta_2)\sigma > \frac{1}{3}\left[\overline{v} + \lambda_3\sigma\right]$,
where $\Delta_1 \equiv (\lambda_1 - \lambda_2)$ and $\Delta_2 \equiv (\lambda_2 - \lambda_3)$. Then $\delta_{seq}^N \equiv \max\{\delta_1, \delta_2\} = \delta_1$ and the collusive allocation **AL1** is optimal.

If assumption **H1** does not hold, then $\max\{\delta_1, \delta_2\} = \delta_2$ and the collusive allocation **AL1** cannot be optimal, that is, it is not optimal to assign bidder 2 on object 1. Consider now the collusive allocation **AL2** in which object 1 is assigned to bidder 3, object 2 to bidder 2, object 3 to bidder 4, and object 4 to bidder 1. The relevant discount factors for bidders 1 and 2

$$\delta_1' \geq \frac{n}{\Pi}\left[2(\lambda_1 - \lambda_2)\sigma + (\lambda_1 - \lambda_3)\sigma\right]$$

and

$$\delta_2' \geq \frac{n}{\Pi}\left[(\overline{v} + \lambda_2\sigma) + (\lambda_2 - \lambda_3)\sigma\right]$$

are obtained by using the fact the bidder 1's optimal deviation takes place on object 1, and bidder 2's optimal deviation takes place on object 3. Notice that

$$\delta_1' - \delta_1 = \frac{n}{\Pi}(\lambda_2 - \lambda_3)\sigma$$

and

$$\delta_2' - \delta_2 = -\frac{n}{\Pi}(\lambda_2 - \lambda_3)\sigma.$$

We can then draw the following conclusion. Assume that **H1** does not hold. Then

1. if $\delta_2' > \delta_1'$, then allocation **AL2** is optimal;

2. if $\delta_2' < \delta_1'$, then allocation **AL2** is optimal if $\delta_1' < \delta_2$, whereas allocation **AL1** is optimal if $\delta_1' > \delta_2$.

6 Discussion

The collusive mechanism described by assumptions **A9-A11** was chosen to highlight in the simplest possible way the "maverick-last" pro-collusive effect of sequential auction, that applies only to split-award collusive agreements, and its interaction with the number of lots and alternative disclosure policies. It is easy to see that all our results not directly relative to the "maverick-last" effect apply to much more general collusive situations. For example, consider the more standard alternative scenario in which **A1-A8** hold but it is the same auction that is infinitely repeated, with a random permutation of the λ_i across the n collusive bidders before each future action-stage-game. In this scenario with changing valuations, the optimal collusive allocation is obviously a rotating one where each period the bidder with the highest valuation, that period's maverick, gets all the objects at the optimal collusive price (marginally higher than the highest fringe bid); and all other bidders submit fake bids and get no object. The optimal punishment is such that if a defection from the rotating scheme takes place, the n bidders revert to non-cooperative bidding forever. In this scenario asmmetries are pooled intertemporally and along the collusive equilibrium path the maverick gets all objects, so that he cannot be "given the last one(s)". Still, the other effects are present. With a fully transparent sequential format a defection is detected after the first lot is stolen, while with the simultaneous format $n-1$ objects can be stole before the others can react. And for the same reasons highlighted in the previous sections partial disclosure policies have little effect in terms of making collusion in sequential auctions harder to sustain.

The focus of the model and its analysis has been on the effects of the sequential auction format on what Comte *et al.* (2002) define the "deviation concern", i.e., on the size of the short term gains of defection from collusive strategies. Under both assumptions **A9-A12** and the alternative scenario sketched in the previous paragraph the "punishment concern", i.e. the strength of the punishment phase disciplining the collusive agreement is not affected by the choice between a simultaneous and a sequential format. In the first scenario the future interaction hence the punishment phase is independent of this choice by assumption. In the second one the static Nash equilibrium of the two auction formats coincides, given the hypothesys of complete information on individual valuations across the n dominant bidders. This last conclusion may change when relaxing the assumption of complete information on valuations across dominant/colluding bidders, and the recent work of Mezzetti *et al.* (2004) suggests it is not at all clear in which direction.

7 Conclusion

Sequential auctions are used to in very different markets. They are arguably simpler to organize than a simultaneous auction when a very large number of objects is sold. However, we have pointed out that the sequential auction possesses two main procollusive features. Whenever a bidding ring aims at reducing the competitiveness of the bidding process by splitting the objects among its members, such a mechanism provides an effective device to monitor the conspirators' adherence to the collusive scheme and to considerably reduce the gains from defection with respect to a simultaneous format. Moreover, the sequential format can, at least partially, solve the ring's problem of reducing the maverick's incentives to defect by allocating to the latter the last object(s) of the sequence.

We have explored whether the seller can counterbalance the procollusive features of the sequential auction by making the mechanism more opaque, but the resuts appear rather discouraging. Witholding part of the information of the outcome of the different rounds of a sequential auction has in general little effect on bidders ability to sustain collusion. In some circumstances the policy of disclosing only the winning price does reduce the ring's monitoring capacity, making collusion somewhat harder to sustain. But the only disclosure policy that has a substantial effect in terms of hindering collusion is the rather extreme one of witholding all information on the outcome of the rounds of the sequentia auction until the very end of the sequence, which renders a sequential auction strategcally equivalent to a simultaneous one.

References

Admati, A.R. and M. Perry, 1991, "Joint Projects without Commitment", *Review of Economic Studies*, 48, pp. 259-276.

Baker J. 2002, "Mavericks, Mergers, and Exclusion: Proving Coordinated Competitive Effects under the Antitrust Laws,"*New York University Law Review*, 77, pp. 135-203.

Blume, A. and P. Heidhues (2003), "Private Monitoring in Auctions," WZB - Wissenschaftszenturm Berlin SP II 2003 - 14, and University of Pittsburgh, *mimeo.*

Compte, O., Jenny, F. and P. Rey (2002), "Capacity Constraints, Mergers and Collusion," *European Economic Review*, **46**, pp. 1-29.

McAfee, P.R. and J. McMillan (1992),"Bidding Rings," *American Economic Review*, **82**, pp. 579-599.

Mezzetti, C., Pekeč, A. and I. Tsetlin (2004), "Sequential vs. Single-Round Uniform-Price Auctions," University of North Carolina, *mimeo.*

Neher, D.V., 1999, "Staged Financing: an Agency Perspective", *Review of Economic Studies*, 66, 255-274.

Smirnov, V. and A. Wait, 2004 "Hold-up and sequential specific investment", *Rand Journal of Economics*, 35(2), 386-400.

Appendix

Proof of Proposition 1.

(i) Consider first the simultaneous format. The collusive device calls bidder i to be active on object i only, $i = 1, \dots, n$. If no defection occurs at the auction stage, each bidder gets an equal share of the (collusive) payoff in the post-action interaction; if, instead, a deviation occurs at the auction stage, interaction in the post-action phase is such that each bidder gets zero profit. Bidder i's IC constraint in the simultaneous action writes then

$$n(\overline{v} + \lambda_i \sigma) \leq (\overline{v} + \lambda_i \sigma) + \delta \frac{1}{n} \Pi \Leftrightarrow \delta \geq n(n-1) \frac{(\overline{v} + \lambda_i \sigma)}{\Pi}.$$

Notice that the resulting incentive constraints are such that bidder 1's constraint is the most binding, whereas bidder n's constraint is the slackest. Collusion is then sustainable in a simultaneous action if and only if

$$\delta \geq n(n-1) \frac{(\overline{v} + \lambda_1 \sigma)}{\Pi} \equiv \delta_{sim}(\sigma). \tag{2}$$

Consider now the sequential auction. Lemma 1 tells us that the collusive device is such that bidder 1 is active on object n, whereas the allocation of all other bidders is irrelevant. Assume then without loss of generality that bidder 2 is allocated to object 1, bidder 3 to object 2,..., bidder n to object $n-1$. The IC constraint for bidder $i = 2, \dots, n$, become

$$2(\overline{v} + \lambda_i \sigma) \leq (\overline{v} + \lambda_i \sigma) + \delta \frac{1}{n} \Pi$$

which yields

$$\delta \geq n \frac{(\overline{v} + \lambda_i \sigma)}{\Pi}. \tag{3}$$

Bidder1's IC constraint in the sequential auction instead writes

$$(\overline{v} + \lambda_1 \sigma) + (n-1)(\lambda_1 - \lambda_2)\sigma \leq (\overline{v} + \lambda_1 \sigma) + \delta \frac{1}{n} \Pi \Leftrightarrow \delta \geq n(n-1) \frac{(\lambda_1 - \lambda_2)\sigma}{\Pi}.$$

Collusion is sustainable at the sequential auction if and only if

$$\delta \geq \max \left\{ n \frac{(\overline{v} + \lambda_2 \sigma)}{\Pi}, n(n-1) \frac{(\lambda_1 - \lambda_2)\sigma}{\Pi} \right\} \equiv \delta_{seq}(\sigma).$$

Notice that assumption **A7** is equivalent to $\overline{v} + \lambda_2 \sigma > (n-1)(\lambda_1 - \lambda_2)\sigma$, which implies that

$$\delta_{seq}(\sigma) = n \frac{(\overline{v} + \lambda_2 \sigma)}{\Pi}.$$

The final step of the proof consists in proving that for all σ, $\delta_{seq}(\sigma) < \delta_{sim}(\sigma)$. It is immediate that

$$n \frac{(\overline{v} + \lambda_2 \sigma)}{\Pi} < n(n-1) \frac{(\overline{v} + \lambda_1 \sigma)}{\Pi}$$

due to **A1** and **A3**.

(ii) Define

$$\delta \geq \delta_{sim}^{(n)}(\sigma;\lambda_n) \equiv n(n-1)\frac{(\overline{v}+\lambda_n\sigma)}{\Pi}$$

the slackest incentive constraint in the simultaneous auction format, that is, the IC constraint of the bidder with the lowest value. We want to prove that there exists a threshold value σ^* such that for all $\sigma < \sigma^*$, $\delta_{seq}(\sigma;\lambda_1,\lambda_2) < \delta_{sim}^{(n)}(\sigma;\lambda_n)$.

Notice that

$$\delta_{seq}(0) = \frac{n\overline{v}}{\Pi} < \frac{n(n-1)\overline{v}}{\Pi} = \delta_{sim}^{(n)}(0).$$

Moreover, assumption **A4** ensures that both

$$\frac{d\delta_{seq}}{d\sigma}(0) = \frac{n\lambda_2}{\Pi},$$

and

$$\frac{d\delta_{sim}^{(n)}(0)}{d\sigma} = \frac{n(n-1)\lambda_n}{\Pi}$$

are bounded away from $\pm\infty$. Since $\delta_{seq}(\cdot)$ and $\delta_{sim}^{(n)}(\cdot)$ are continuous functions, there must exists a value σ^* such that for all $0 < \sigma < \sigma^*$, $\delta_{seq}(\sigma) < \delta_{sim}^{(n)}(\sigma)$. ■

Proof of Proposition 3.

i) When $\sigma = 0$, bidder i's valuation of each lot is $\frac{\overline{v}}{k}$. The most collusive device (conditional on each bidder getting at least one lot) is such that each bidder is allocated exactly k lots. Thus the relevant incentive compatibility constraint writes

$$nk\frac{\overline{v}}{k} \leq k\frac{\overline{v}}{k} + \delta\frac{\Pi}{n} \Leftrightarrow \delta \geq n(n-1)\frac{\overline{v}}{\Pi},$$

which is independent of k.

ii) Suppose now that $\sigma > 0$. In this case, the most collusive allocation (conditional on each bidder getting at least one lot) is the one that assigns m_i^* lots to bidder i in order to minimize the most binding incentive constraint ($m_i^* \geq 1$). Thus we define

$$M^*(k) = (m_1^*(k), \ldots, m_n^*(k)),\ m_i \geq 1$$

the allocation of lots that solves

$$\min_{(m_1(k),\ldots,m_n(k))} \{\max\{\delta(m_1(k)), \ldots, \delta(m_n(k))\}\}.$$

Suppose that

$$\delta_j(k) = \delta(m_j^*(k)) = \max\{\delta(m_1^*(k)), \ldots, \delta(m_n^*(k))\},$$

that is, at the most collusive allocation bidder j's incentive constraint is the relevant constraint for the sustainability of collusion. Notice that

$$\delta_j(k) = \frac{n}{\Pi}(nk - m_j^*(k))\frac{\overline{v} + \lambda_j\sigma}{k} = (n - \frac{m_j^*(k)}{k})(\overline{v} + \lambda_j\sigma).$$

Consider now a number of lots $k' > k$. The allocation $M^*(k')$ must be such that bidder j is allocated *at least* $m_j^*(k)$ lots. However, a higher number of lots enlarges the set of feasible allocations, which implies that $\frac{m_j^*(k')}{k'} \geq \frac{m_j^*(k)}{k}$. Thus

$$\max\{\delta(m_1^*(k')), \ldots, \delta(m_n^*(k'))\} \leq \max\{\delta(m_1^*(k)), \ldots, \delta(m_n^*(k))\}$$

Finally, when k grows arbitrarily large the fraction of lots optimally allocated to any bidder i, $\frac{m_i^*(k)}{k}$, is bounded by 1. Thus,

$$\lim_{k\to\infty} \max\{\delta(m_1^*(k)), \ldots, \delta(m_n^*(k))\} > \frac{n}{\Pi}(n-1)(\overline{v} + \lambda_n\sigma) > 0.$$

■

Proof of Proposition 4. Consider any coordination device in which bidder i gets *at least* $m_i \geq 1$ lots. Notice that bidder 1's individual rationality constraint (**A7**) can be rewritten as follows

$$m_1\frac{\overline{v} + \lambda_1\sigma}{k} > nk\frac{(\lambda_1 - \lambda_2)\sigma}{k}.$$

The IC constraint for bidder $j \in \{2, \ldots, n\}$ in the sequential auction writes

$$(m_j + 1)\frac{(\overline{v} + \lambda_j\sigma)}{k} \leq m_j\frac{\overline{v} + \lambda_j\sigma}{k} + \delta\frac{\Pi}{n} \Leftrightarrow \delta \geq \frac{n}{\Pi}\frac{\overline{v} + \lambda_j\sigma}{k}.$$

which does not depend on m_j. It is immediate that

$$\lim_{k\to\infty} \frac{n}{\Pi}\frac{\overline{v} + \lambda_j\sigma}{k} = 0, \ \forall j \in \{2, \ldots, n\}.$$

Consider now bidder 1's IC constraint. We have that

$$\begin{aligned}
\frac{\overline{v} + \lambda_1\sigma}{k} + (nk-1)\frac{(\lambda_1 - \lambda_2)\sigma}{k} &\leq m_1\frac{\overline{v} + \lambda_1\sigma}{k} + \delta\frac{\Pi}{n} \Leftrightarrow \\
\delta &\geq \frac{n}{\Pi}\left[(nk-1)\frac{(\lambda_1 - \lambda_2)\sigma}{k} - (m_1 - 1)\frac{\overline{v} + \lambda_1\sigma}{k}\right] \Leftrightarrow \\
\delta &\geq \frac{n}{\Pi}\left[(n - \frac{1}{k})(\lambda_1 - \lambda_2)\sigma - (\frac{m_1}{k} - \frac{1}{k})(\overline{v} + \lambda_1\sigma)\right].
\end{aligned}$$

Thus

$$\begin{aligned}
& \lim_{k\to\infty}\left[(n - \frac{1}{k})(\lambda_1 - \lambda_2)\sigma - (\frac{m_1}{k} - \frac{1}{k})(\overline{v} + \lambda_1\sigma)\right] \\
< & \lim_{k\to\infty}\left[(n - \frac{1}{k})(\lambda_1 - \lambda_2)\sigma - (n\frac{(\lambda_1 - \lambda_2)\sigma}{\overline{v} + \lambda_1\sigma} - \frac{1}{k})(\overline{v} + \lambda_1\sigma)\right] \\
= & \ 0,
\end{aligned}$$

that is, at the limit, bidder 1's IC constraint is always satisfied. ■

Proof of Propostion 6.

(i) It suffices to prove that the gains from deviation in both formats are bigger when synergies are positive than in the framework without synergies, whereas the gains from cooperation remain unchanged. Consider first the simultaneous format. Bidder i's IC constraint can be written as in (1), that is,

$$\Pi_{sim}^{D_i^\alpha} \leq \Pi_{sim}^{C_i^\alpha} + \delta\frac{\Pi}{n},$$

where

$$\Pi_{sim}^{D_i^\alpha} = n(\overline{v} + \lambda_i\sigma)(1+\alpha) > n(\overline{v} + \lambda_i\sigma) = \Pi_{sim}^{D_i}$$

and

$$\Pi_{sim}^{C_i^\alpha} = \Pi_{sim}^{C_i} = (\overline{v} + \lambda_i\sigma) + \delta\frac{\Pi}{n}.$$

Thus the presence of positive synergies raises the gains of the optimal defection but leaves unchanged bidder i's payoff form adhering to the collusive allocation of the objects. Consequently, the value of the discount factor that makes collusion sustainable must be higher when there are positive synergies than when synergies are absent, that is,

$$\delta \geq \delta_{sim}^\alpha \equiv \frac{n}{\Pi}\left[(n-1)(\overline{v} + \lambda_1\sigma)(1+\alpha)\right] > \frac{n}{\Pi}\left[(n-1)(\overline{v} + \lambda_1\sigma)\right] \equiv \delta_{sim}.$$

Consider now the sequential format. Notice that the presence of synergies does not alter the optimality of the collusive allocation in which the highest-value bidder gets object n and bidder $i = 2, \ldots, n$ gets objects $i-1$. We have that for bidder $i = 2, \ldots, n$

$$\Pi_{seq}^{D_i^\alpha} = 2(\overline{v} + \lambda_i\sigma)(1+\alpha) > 2(\overline{v} + \lambda_i\sigma) = \Pi_{seq}^{D_i},$$

whereas for bidder 1

$$\Pi_{seq}^{D_1^\alpha} = n\left[(\overline{v} + \lambda_1\sigma)\right](1+\alpha) - (n-1)(\overline{v} + \lambda_2\sigma) > (\overline{v} + \lambda_1\sigma) + (n-1)(\lambda_1 - \lambda_2)\sigma = \Pi_{seq}^{D_1}.$$

Since for each bidder $i = 1, \ldots, n$

$$\Pi_{seq}^{C_i^\alpha} = \Pi_{seq}^{C_i} = (\overline{v} + \lambda_i\sigma),$$

it results that the value of the discount factor making collusion sustainable is strictly greater when synergies are positive than when they are absent, that is,

$$\begin{aligned}
\delta \geq \delta_{seq}^\alpha &\equiv \frac{n}{\Pi}\max\left\{(n-1)\left[(\lambda_1 - \lambda_2)\sigma + (\overline{v} + \lambda_1\sigma)\alpha\right], \left[(\overline{v} + \lambda_2\sigma)(1+2\alpha)\right]\right\} \\
&> \frac{n}{\Pi}(\overline{v} + \lambda_2\sigma) \\
&\equiv \delta_{seq}.
\end{aligned}$$

(ii) We have that

$$\begin{aligned}\frac{\Delta^{\alpha}(\cdot)}{d\,\alpha} &= \frac{n}{\Pi}\left[\max\{(n-1)(\overline{v}+\lambda_1\sigma),2(\overline{v}+\lambda_2\sigma)\}-(n-1)(\overline{v}+\lambda_1\sigma)\right] \\ &= \frac{n}{\Pi}\left[(n-1)(\overline{v}+\lambda_1\sigma)-(n-1)(\overline{v}+\lambda_1\sigma)\right] \\ &= 0,\end{aligned}$$

(iii) This simply follows from noticing that package bidding affects neither the value of $\Pi_{sim}^{D_i^{\alpha}}$ nor $\Pi_{sim}^{C_i^{\alpha}}$. Thus it does not modify δ_{sim}^{α} either. ■

Proof of Proposition 8.

i) Assume $\sigma = 0$. A simple 'contagion' argument will show that the sustainability of collusion is unaffected by either the "PA" or the "ND" policy. To see this, recall that bidders' collusive allocation becomes irrelevant when they are perfectly symmetric. Consider then the allocation in which bidder i is awarded object i, $= 1, \ldots, n$. Suppose that bidder i deviates by becoming active on object $j \neq i$. Under the "PA" policy, bidder i's defection goes undetected to all other bidders $k \neq i, j$ only if bidder i submits a bid exactly equal to the collusive price. Since this strategy significantly increases the risk of being fined by the Antitrust Authority, any defection must consist in submitting a price different from the collusive one and is immediately detected. Thus the "PA" policy does not affect the sustainability of collusion with respect to case of full transparency.

Under the "ND" policy, bidder i's optimal deviation on object j consists in submitting a bid marginally higher than the collusive price (i.e., ν) since it yields (almost) the same payoff but it does not increase the probability of a fine being levied by the Antitrust Authority. Although the deviation never becomes public information, Nash reversion on object $j+1$ implies that bidder $j+1$ will revert to competitive bidding on object $j+2$, bidder $j+2$ will revert to competitive bidding on object $j+3$ etc. Thus the gains from defection are equal to $\overline{v}$, and coincide with those under a fully transparent sequential auction.

ii) Assume $\sigma > 0$. Under the "PA" policy, any defecting bidder must submit a bid different from the collusive one, so any deviation is immediately detected. This shows that $\delta_{seq}^{P}(\sigma) = \delta_{seq}(\sigma)$, $\forall \sigma > 0$.

Consider now the "ND" policy. We have to show that the relevant incentive compatibility constraint under the "ND" policy is more binding than under a policy of full transparency. To this end, recall that in the latter case the relevant incentive constraint for collusion to be sustainable is bidder 2's. Given that bidder 2's gains from cooperation coincide under the two disclosure policies, we have to show that bidder 2's gains from defection cannot be any lower under the "ND" policy than under full disclosure.

Define $\Psi_{seq}^{D_2}(i)$ bidder 2's gains from defection in the sequential auction when *a)* the seller adopts the "ND" policy, and *b)* the collusive agreement assigns bidder 2 to object $i = 1, \ldots, n$.

Recall from lemma 1 that $\Pi^{D_2}_{seq} = 2(\overline{v} + \lambda_2\sigma)$ represents bidder 2's gains from defection in the sequential auction under the assumption of full transparency. We have to distinguish three cases.

case 1. Consider *any* collusive agreement that assigns bidder 2 to object $i = 1, \ldots, n-2$, and bidder 1 to object n. Then

$$\Psi^{D_2}_{seq}(i) \geq 2(\overline{v} + \lambda_2\sigma) + (n - i - 1)(\lambda_2 - \lambda_3)\sigma > 2(\overline{v} + \lambda_2\sigma) = \Pi^{D_2}_{seq},$$

where the lower bound for $\Psi^{D_2}_{seq}(i)$ is derived by considering the worst possible allocation (from bidder 2's perspective) in which bidder 3 is allocated to object $i+1$ and bidder 2 becomes active on the same object. Indeed, upon observing a deviation on object $i+1$, bidder 3 does not know the identity of the defecting bidder. We assume that bidder 3 holds that a deviation always comes form a bidder with a higher valuation (call them *pessimistic beliefs*). This implies that Nash reversion from round $i+2$ onwards generates a stream of selling prices each of them at least equal to $\overline{v} + \lambda_3\sigma$ and thus the lower bound to the value of a deviation to bidder 2. Notice also that

$$\Psi^{D_1}_{seq}(n) \geq (\overline{v} + \lambda_1\sigma) + (n - 1)(\lambda_1 - \lambda_2)\sigma = \Pi^{D_1}_{seq},$$

that is, bidder 1's gains from deviation cannot be lower under the "ND" policy than under full disclosure. It must then be the case that the relevant incentive constraint for collusion - bidder 2's constraint - becomes more stringent than under a policy of full disclosure, which implies $\delta^N_{seq}(\sigma) > \delta_{seq}(\sigma)$, $\forall \sigma > 0$.

case 2. Consider now *any* collusive agreement that allocates object $n-1$ to bidder 2, and object n to bidder 1. We have that

$$\Psi^{D_2}_{seq}(n-1) = \max\{(\overline{v} + \lambda_2\sigma) + g(a_j; \lambda_2), 2(\overline{v} + \lambda_2\sigma)\} \geq \Pi^{D_2}_{seq},\ j = 3, \ldots, n.$$

By deviating from the collusive agreement and becoming active on object n, bidder 2 can guarantee to herself a payoff of $2(\overline{v} + \lambda_2\sigma) = \Pi^{D_2}_{seq}$. However, such a set of collusive schemes enlarges bidder 2's space of possible deviations. Bidder 2 can become active on object 1, thus getting $\overline{v} + \lambda_2\sigma$, and the amount of non-cooperative profit, $g(a_j, \lambda_2)$, which depends upon the allocation scheme of all other bidders, a_j, $j = 3, \ldots, n$. However, if bidder 3 is allocated to object 1

$$\Psi^{D_1}_{seq}(n) \geq (\overline{v} + \lambda_1\sigma) + (n - 1)(\lambda_1 - \lambda_3)\sigma > \Pi^{D_1}_{seq}.$$

and bidder 1's incentive constraint becomes the most binding. Assumption **A13** allows then to conclude that the relevant incentive constraint (bidder 1's) becomes more stringent than under a policy of full disclosure, so $\delta^N_{seq}(\sigma) > \delta_{seq}(\sigma)$, $\forall \sigma > 0$.

case 3. Lastly, consider *any* collusive allocation in which bidder 2 is assigned to object n and bidder 1 is assigned to object $i = 1, \ldots, n-1$. By deviating on object n, bidder 1 can guarantee to herself $\Psi_{seq}^{D_1}(n-1) = 2(\overline{v} + \lambda_1 \sigma)$. Thus bidder 1's threshold value of the discount factor

$$\delta_1^N(\sigma) \geq \frac{n}{\Pi}(\overline{v} + \lambda_1 \sigma) > \delta_{seq}(\sigma), \forall \sigma > 0.$$

That is, by allocating bidder 1 to object $n-1$, the relevant discount factor for collusion becomes strictly higher than under a policy of full disclosure because of the sub-optimal assignment of bidder 1. Thus we conclude that $\delta_{seq}^N(\sigma) \geq \delta_{seq}(\sigma)$, $\forall \sigma > 0$.

To complete the proof we have to show that none of the bidders who have observed a deviation would be strictly better off by informing all other bidders. Suppose that a deviation takes place on object i that had been assigned to bidder j by the collusive scheme. It is immediate that the defecting bidder is strictly better off by not informing all other bidders. Remaining silent minimizes the set of competitors on the remaining objects. Consider now bidder j, that is, the one who should have obtained object i. Bidder j's behavior will depend upon her beliefs about the identity of the defecting bidder. Under 'pessimistic beliefs' defined above, bidder j expect zero profit from the reversion to competitive bidding on object $i+1$. Thus she will be indifferent between informing and not informing all other bidders about the defection. If, instead, bidder j holds that a defection may have come from a lower valuation opponent, then she is strictly better off by not informing all other bidders since, in this case, she may get strictly positive profit from the reversion to competitive bidding on object $j+1$. ■