

# The Overconfidence Problem in Insurance Markets\*

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December 2004

## Abstract

Adverse selection has long been recognized as a rationale for government intervention in insurance markets and for the adoption of public compulsory insurance. A different rationale for compulsory insurance is that overconfident individuals may underinsure because they underestimate the relevant risks. We show that government intervention is not a Pareto improvement in an adverse selection model with a significant fraction of overconfident agents. We underline that behavioral biases need not be the basis for government intervention. In fact, behavioral biases may overturn existing compelling reasons for intervention in the economy. Our model also delivers novel positive implications on aggregate variables that have been at the center of recent empirical investigation.

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\*We thank Guillaume Frechette, Hide Hichimura, Ben Lockwood, Costas Meghir, Lars Nesheim and Jean Tirole for their comments.

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*The really risky clients are those who believe they are first-class drivers.*

CEO of a major insurance company, as quoted in Chiappori and Salanié (2000), page 73.

## 1 Introduction

Economists and policy-makers have long debated the merits of government intervention in insurance markets. In the presence of asymmetric information, compulsory public insurance (e.g. Social Security and Medicare in the U.S.) may result in a Pareto improvement.<sup>1</sup> A different rationale for compulsory public insurance is that some individuals underinsure because they underestimate the relevant risks.<sup>2</sup> Compulsory insurance, the argument goes, should be imposed on overconfident individuals for their own better sake.

It is well documented that a significant fraction of individuals overestimate their health, financial planning and driving ability, and that this often results in underinvestment in precautionary activities.<sup>3</sup> However, the full implications of overconfidence in insurance markets are far from clear. In particular, the supposition that overconfidence is a supporting factor for government intervention has not yet been investigated in the context of a developed model of insurance.

We consider the welfare implications of several policies in the presence of overconfidence. To keep matters as transparent as possible, we build on the well-known insurance markets model of Rothschild and Stiglitz (1976) with adverse selection. Like Rothschild and Stiglitz (1976), we assume that insurance companies are perfectly competitive and cannot directly observe their customers' risk. Unlike Rothschild and Stiglitz (1976), we allow for overconfident agents. Some agents believe that their risk is low, when, in fact, it is high. The other agents know their risk.

We confirm that overconfident agents may underinsure in the misperception that their insurance price is too high.<sup>4</sup> However, we also show that, when the economy has a significant fraction

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<sup>1</sup>This argument, already suggested by Rothschild and Stiglitz (1976), was fully demonstrated by Wilson (1977) and Dahlby (1983). This insight is highlighted in public economics textbooks (e.g. Auerbach and Feldstein, 2002), and often appears on institutional debates, see Mark Pauly (1994) on the proposal for the “Health Care Reform and Health Security Act” presented by President Clinton to Congress.

<sup>2</sup>For example, in presenting his proposals on the implementation of universal public health care, and referring to the disturbingly large number of uninsured individuals in the U.S., Peter Diamond's 1992 Econometric Society Presidential Address quotes “Except for a few totally unable to purchase insurance [...] it is natural to say that people are without insurance because it costs more than it appears to be worth to them [...] It seems useful to divide the population without insurance into three groups. *Some are without insurance because they misperceive the risks or consequences of this decision.*”, Diamond (1992), page 1236.

<sup>3</sup>We discuss the evidence for overconfidence in section 2.

<sup>4</sup>An estimated 15.2 percent of the population, or 43.6 million people, had no health insurance coverage in

of overconfident agents, compulsory insurance does not Pareto improve upon the laissez-faire equilibrium. It increases equilibrium prices and makes low risk agents worse off. Given that, in the absence of overconfidence, compulsory insurance may be Pareto Improving, the basic argument of overconfidence as a rationale for compulsory insurance is overturned. Overconfidence runs counter the established adverse-selection rationale for compulsory insurance.

Overconfidence changes the equilibrium of adverse-selection models qualitatively. Without overconfidence, the equilibrium is pinned down by a binding incentive-compatibility constraint. Low risk agents' insurance is constrained to ensure separation from high risk subscribers. Compulsory insurance benefits high risk agents directly because a contract designed for all agents has better terms than a contract designed for high risk agents alone. Compulsory insurance also benefits low risk agents because it relaxes the incentive compatibility constraint imposed by high risk agents. On the other hand, when the economy has a significant fraction of overconfident agents, the incentive-compatibility constraint no longer binds in equilibrium. Compulsory insurance is thus equivalent to a transfer of wealth from low risk to high risk agents.<sup>5</sup>

The incentive-compatibility constraint does not bind in equilibrium when the fraction of overconfident agents is significant because overconfident agents cannot be screened from low risk agents (they share the same beliefs about their risk). The higher the fraction of overconfident agents in the economy, the higher the risk of the pool of low risk and overconfident agents and the higher the price insurance firms must offer to avoid negative profits. At high prices, these contracts are unattractive to high risk agents. Hence, the incentive-compatibility constraint does not bind.

Our basic result goes beyond compulsory insurance. We show that government intervention cannot Pareto improve upon the laissez-faire equilibrium by means of any general mechanism, unless it changes the fraction of overconfident agents in the economy. This result holds regardless of whether the social planner maximizes perceived or actual welfare. When the incentive-compatibility constraint does not bind, by revealed preferences, the equilibrium outcome is optimal for low risk agents. Hence, state intervention altering this outcome makes low risk agents worse

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2002, according to the US Census report released on September 30, 2003. According to the data published by the Insurance Research Council, an average of 14.9% of drivers was not insured in the period from 1989 to 1997.

<sup>5</sup>In the context of automobile insurance our analysis applies only to personal loss insurance. Compulsory automobile insurance was initially introduced in the US only in the form of liability insurance, to ensure financial assistance to accident victims when drivers at fault have limited assets. Nowadays, however, many States require mandatory insurance for personal loss in the forms of Personal Injury Protection and Uninsured Motorist insurance (see the *Summary of Selected State Laws* published by American Insurance Association, 1976-2003).

off.

Our model shows that bringing together the literatures of behavioral economics and mechanism design may provide a better understanding of policy implications under behavioral assumptions. Contrary to *prima facie* intuition, behavioral biases need not be the basis for paternalistic policies. This follows because government interventions that seemingly counteract the economic effects of biases such as overconfidence may turn out to be counterproductive. On the other hand, policies that directly reduce overconfidence in the economy do make everybody better off, provided that their costs are sufficiently low.<sup>6</sup> In the context of automobile insurance, such policies materialize in voluntary training programs, designed by practitioners in risk and accident prevention, to help drivers improve their self-assessment skills.<sup>7</sup> In equilibrium, low risk and overconfident drivers join these programs. Low risk drivers' self-assessment skills are not improved, but they indirectly benefit from joining programs because they earn a reduction in insurance price.

## 1.1 Positive Analysis of Overconfidence in Insurance Markets

In addition to our welfare analysis, we also offer some simple positive results that distinguish our analysis from most previous models of overconfidence and from most insurance models that abstract from overconfidence. First, we find that overconfidence yield different implications from a reduction of risk aversion. Adding a small fraction of risk neutral individuals to the basic Rothschild and Stiglitz (1976) model reduces average equilibrium insurance coverage.<sup>8</sup> Instead, adding a small fraction of overconfident individuals increases equilibrium coverage. This is unlike most models of overconfidence, where both overconfidence and reduction in risk aversion lead to greater willingness to take risks.<sup>9</sup>

Second, our model accounts for the following two stylized facts, which are seldom explained

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<sup>6</sup>This result also holds for both the public's perceived and actual welfare.

<sup>7</sup>On a related note, Benabou and Tirole (2003) characterize and discuss several strategies that an individual may use to manipulate her own or her counterparts' self-confidence.

<sup>8</sup>This result holds when the fraction of overconfident agents is small, and hence the incentive-compatibility constraint binds in equilibrium. Adding overconfident agents to the economy increases the equilibrium price of low-risk and overconfident agents' contracts. As these contracts become less attractive to high-risk subscribers, low-risk and overconfident agents' insurance needs to be constrained less to ensure separation. This results in an increment of aggregate insurance coverage.

<sup>9</sup>Among the papers investigating the economic consequences of overconfidence, Benabou and Tirole (2002) show that an overconfident time-inconsistent individual may strategically choose to ignore information about her uncertain payoff, and Compte and Postlewaite (2003) show that it may be optimal to be overconfident when performance is enhanced by confidence, even though this may result in taking excessive risks.

simultaneously in previous models of insurance. For any given coverage amount, the prices of insurance contracts are higher for agents who belong to riskier classes.<sup>10</sup> Yet the data also show large heterogeneity in prices within risk classes (e.g. Chiappori and Salanié, 2001). In our model, agents with different perceived risk choose different contracts, within each risk class. Unlike most models of insurance with symmetric information, prices differ within risk classes. But unlike the Rothschild and Stiglitz (1976) model, the agents' actual risk cannot be fully signaled, because overconfident and low risk agents choose the same contract. The price of this contract is higher for agents in riskier classes, because the fraction of overconfident agents is higher.

Finally, we study implications for insurance coverage. These results distinguish our model from most models of insurance with symmetric information, and from most asymmetric information models that abstract from overconfidence. A general and robust implication of asymmetric information (without overconfidence) is a positive relationship between ex-post risk and insurance coverage, within each risk class (Chiappori et al., 2002). Recent empirical analyses reject this implication. A statistically non-significant relation is detected by Chiappori and Salanié (2001) in French automobile insurance datasets, by Cawley and Philipson (1999) in U.S. life insurance datasets, and by Cardon and Hendel (2001) in the 1987 National Medical Expenditure Survey.<sup>11</sup>

We extend our model and assume that overconfident individuals are riskier than high risk unbiased individuals. We show that the relation between insurance coverage and ex-post risk becomes indeterminate. This does not mean that overconfidence is observationally equivalent to symmetric information, because the relationship between insurance coverage and self-reported risk remains positive. Furthermore, symmetric information and perfect competition imply that all agents are fully insured. In our model, overconfident and low-risk individuals may be severely underinsured even under perfect competition.

The paper is presented as follows. After the literature review, section 3 presents the model. A graphical description of equilibrium is presented in section 4. Sections 5 and 6 informally present our normative and positive results. Section 7 concludes. The appendix lays out the formal analysis.

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<sup>10</sup>A risk class is a set of agents with observationally equivalent risk.

<sup>11</sup>On the other hand, Cohen (2003) found a positive relationship in an Israeli cross-sectional data set for older drivers (but not for young novice drivers).

## 2 Experimental Evidence of Overconfidence

The experimental evidence on overconfidence is overwhelming. Here we mention only a small subset of this literature. The interested reader may consult a survey (available upon request) that we compiled on the topic.

Survey studies, with the most disparate subject samples, show that a large fraction of individuals believe that they are healthier, more financially secure and better drivers than the median individual.<sup>12</sup> Experimental studies find evidence of overconfidence by comparing self-reported risk with objective risk. Kreuter and Strecher (1995) and Robb et al. (2004) found evidence of health risk underestimation in relation to medical exams. Overconfidence has been detected by Groeger and Grande (1996) who compare drivers' self-assessments skills with those assessed by an instructor. Walton and McKeown (2001) compare self-reported and actual speed of drivers. Hoch (1985) found that MBA students overestimate the number of job offers they will receive and the magnitude of their salary.<sup>13</sup> These findings have been widely replicated, and we are not aware of a single empirical study that in any way found direct evidence against overconfidence.

The implications of overconfidence on precautionary behavior have been confirmed in several studies. Overconfidence has been recognized as a major determinant of traffic safety in many institutional studies (e.g. the European Union projects by Hatakka et al., 2002, and by Bartl, 2000). Health risk underestimation is recognized as a major barrier preventing healthy behavior (see the survey by Hoorens, 1994). There is also evidence that overconfidence induces poor financial planning. Benartzi (2001) finds that employees severely underestimate the risks of their own company stock, which is over-represented in their retirement saving plans.

To our knowledge, empirical testing of overconfidence in insurance markets is still underdeveloped. However, there are some studies on the subject. Spurred by proposed welfare reforms in the UK, Cebulla (1999) conducted surveys on the perception of the risk of becoming unemployed

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<sup>12</sup>Such results by Svenson (1981) on overconfidence of driving ability in Sweden have been replicated in Australia, the United States, Canada, Britain, Finland, France, as well as in Germany, Spain and Brazil. The results by Weinstein (1980) that subjects overestimates of their future financial success have been replicated in the US, Sweden, New Zealand, Belgium, Morocco, Poland, the UK, Hawaii, Switzerland, and in the Netherlands. Health overconfidence has been detected in samples from the US, the UK, the Netherlands, Israel, Tanzania, and Norwegia.

<sup>13</sup>There is also strong specific evidence that overconfidence does not vanish with learning nor with experience. For example, Dalziel and Job (1997) found that professional drivers, such as metropolitan taxi drivers from Sydney, underestimate their risk of automobile accident.

and the willingness to purchase unemployment insurance. He detected underestimation of risk by comparing self-reported assessments with statistical assessments. Risk underestimation reduced the willingness to buy insurance. Bhattacharya, Goldmanz and Sood (2004) study secondary life-insurance markets, where consumers with a life-threatening illness may sell their life insurance policies in return for an up-front payment. They find evidence that patients who underestimate their risk of death are unwilling to hold insurance coverage.<sup>14</sup>

We conclude this brief review with an important qualification. While there is strong evidence that subjects underestimate risk on activities that they believe are under their control, such as driving or financial planning, or that pertain to their self-image, such as health, there is no empirical evidence (to our knowledge) that subjects underestimate the risk of other uncertain events such as fires, floods, earthquakes, theft, malfunctioning of durable goods etc. Hence our analysis does not apply to such insurance markets. Among the experimental papers studying some of these markets, some suggest that subjects overinsure (e.g. Eisner at Strotz, 1961, on airplane travel insurance) and some that they underinsure (e.g. Kuhnreuter et al., 1978, on disaster insurance).

The aim of this paper is not to build a complete behavioral theory of insurance. We focus on implications of a well-documented and specific bias: underestimation of personal risk. We assume that subjective probabilities differ from objective ones, but our analysis is entirely within the standard expected utility representation. Among further departures from standard insurance models, one may consider prospect theory (Kahneman and Tverski 1979), and regret theory (Bell 1982). Our analysis can be extended by employing some of these non-standard theory utility representation, instead of the expected utility representation. Preliminary investigations suggest that our main results remain qualitatively unchanged (details available upon request).

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<sup>14</sup>Beyond such empirical analyses, there seems to be consensus that underestimation of health risks is one of the factors explaining the large number of individuals without insurance in the U.S. (an estimated 15.2 percent of the population in 2002, or 43.6 million people). Also, the very limited purchase of long-term care insurance the U.S. (roughly 10% of those aged 65 purchased such insurance in 2000), may be partly be blamed on the public underestimation of the risk involved in being uninsured.

### 3 The Model

Our model introduces overconfidence in the basic framework of competitive insurance markets with adverse selection by Rothschild and Stiglitz (1976). For each agent, there are two possible states of the world: in state 2 an accident of damage  $d$  occurs and the individual's wealth is  $W - d$ , and in state 1 her wealth is  $W$ . An insurance contract is a pair  $\alpha = (\alpha_1, \alpha_2)$  so that the individual's wealth is  $(W - \alpha_1, W - d + \alpha_2)$  when buying  $\alpha$ . The amount  $\alpha_1$  is the *premium*,  $\alpha_1 + \alpha_2$  is the payment, or *insurance coverage*, and  $P = \alpha_1/(\alpha_1 + \alpha_2)$  is the *price* of a unit of insurance. We assume that  $\alpha_1 \geq 0$ ,  $\alpha_2 \geq 0$ : individuals cannot take on more risk through an insurance contract. Let  $p$  be the likelihood that the accident occurs. So, an agent's expected utility is  $V(W, d; p, \alpha) = (1 - p)U(W - \alpha_1) + pU(W - d + \alpha_2)$ . We assume that  $U$  is twice differentiable, that  $U' > 0$  and that  $U'' < 0$ , so that individuals are risk averse. Risk is measured by the probability  $p$  of an accident. It can either be high ( $p_H$ ) and low ( $p_L$ ), with  $p_H > p_L$ . There are three types of agents in the economy. *High risk* (type  $H$ ) and *Low risk* (type  $L$ ) are agents who know that their risks are  $p_H$  and  $p_L$ , respectively. *Overconfident* (type  $O$ ) agents believe that their risk is low when in fact it is high.<sup>15</sup> Let  $\lambda \in (0, 1)$  be the fraction of low risk agents in the economy. Let  $\kappa \in [0, 1]$  the fraction of overconfident agents in the economy, so that  $\kappa + \lambda \leq 1$ .

The insurance market is a competitive industry of expected profit maximizing (risk neutral) companies. A contract  $\alpha$  sold to an agent with risk  $p$  yields expected profit  $\pi(p, \alpha) = (1 - p)\alpha_1 - p\alpha_2$ . The insurance firms cannot observe a subscriber's risk or beliefs, but they know  $\kappa$  and  $\lambda$ . A *perfectly-competitive equilibrium* is a set of contracts  $A$  such that: (i) no contract  $\alpha \in A$  makes strictly negative expected profits, and (ii) no contract  $\alpha' \notin A$  makes strictly positive profits.

**Remark.** As in Rothschild and Stiglitz (1976), a perfectly-competitive equilibrium may fail to exist for some parameter values. Hence, we also consider the weaker concept of locally-competitive equilibrium, which always exists and is formally defined in Appendix A. The parameter values of our main interest are those for which a perfectly-competitive equilibrium does exist.

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<sup>15</sup>To simplify the exposition, we focus on the case that the difference between low risk and high risk is not too small relative to the damage  $d$ . That is, we assume that

$$\frac{(1 - p_L)/p_L}{(1 - p_H)/p_H} > \frac{U'(W - d)}{U'(W)}.$$



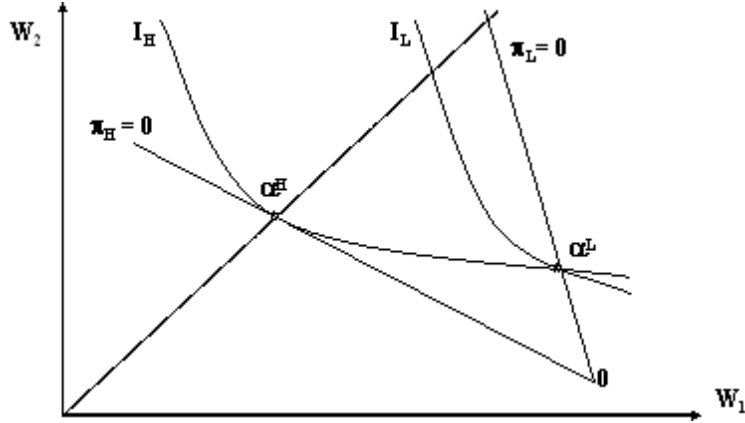


Figure 1: Equilibrium without Overconfidence

## 4 Graphical Description of Equilibria

### 4.1 Equilibrium in Insurance Markets without Overconfidence

In order to highlight the effect of overconfidence in insurance markets, we briefly consider the model without overconfidence. That is, we assume that  $\kappa = 0$ . Rothschild and Stiglitz (1976) show that the equilibrium is separating. Subscribers are screened according to the contract they choose. High-risk individuals fully insure. Their contract  $\alpha^H$  equalizes wealth across states. So, this contract can be found in the intersection of the 45-degree line with the zero-profit line  $\pi_H = 0$ . Incentive compatibility requires that high risk subscribers (weakly) prefer contract  $\alpha^H$  to the low-risk individuals' contract  $\alpha^L$ . Hence, the contract  $\alpha^L$  can be found in the intersection of the zero-profit line  $\pi_L = 0$  with the indifference curve  $I_H$  (through the high risk agents' contract  $\alpha^H$ ). The contracts  $(\alpha^L, \alpha^H)$  are a (unique) perfectly-competitive equilibrium as long as the fraction  $\lambda$  of low-risk subscribers is sufficiently small. The equilibrium contract is illustrated in Figure 1.

### 4.2 Equilibrium in Insurance Markets with Overconfidence

In this section we present a graphical description of the equilibrium with overconfidence (i.e., we assume that  $\kappa > 0$ ). To simplify the exposition, we focus on equilibrium characterization. Conditions for perfectly-competitive equilibrium existence are presented in Appendix A.<sup>16</sup>

<sup>16</sup>We find that there is no perfectly-competitive equilibrium if the fraction of low-risk agents  $\lambda$  is larger than a threshold  $\lambda_0(\kappa)$ . Then, the equilibrium we characterize is only a locally competitive equilibrium.

The core of our analysis is based on two intuitive insights. The first one is that *insurance firms cannot screen between overconfident and low-risk individuals* because, at the time of insurance purchase, both types believe that their risk is low. Given this qualification, basic arguments, analogous to the analysis of Rothschild and Stiglitz (1976), allows us to conclude that, in equilibrium, high-risk individuals purchase a contract  $\alpha^H$ , whereas low-risk and overconfident individuals choose a different contract  $\alpha^{LO}$ . As in the case without overconfidence, high risk individuals fully insure.

The average accident probability of overconfident and low-risk individuals is

$$p_{LO} \equiv \frac{\kappa p_H + \lambda p_L}{\kappa + \lambda}.$$

Perfect competition requires that the equilibrium contract  $\alpha^{LO}$  satisfies the zero-profit condition  $(1 - p_{LO})\alpha_1^{LO} - p_{LO}\alpha_2^{LO} = 0$  (in short,  $\pi_{LO} = 0$ ), that is to say, the price of insurance  $P^{LO}$  coincides with  $p_{LO}$ . As the fraction of overconfidence agents  $\kappa$ , increases, the zero-profit line  $\pi_{LO} = 0$  rotates counterclockwise towards the zero-profit line for high risk types,  $\pi_H = 0$ .

This leads to the second insight. Unlike in the case without overconfidence, *incentive compatibility need not be binding in equilibrium*. As we argue below, it does not bind when the fraction of overconfident individuals  $\kappa$  is large enough.

In order to describe the equilibrium in the presence of overconfidence, we must distinguish between three different cases depending on the parameters  $\kappa$  and  $\lambda$ . The three significant parameter regions are characterized by the threshold functions  $\kappa_1(\lambda)$  and  $\kappa_2(\lambda)$  formally defined in the appendix. In case 1, the fraction of overconfident agents  $\kappa$  is small relative to the fraction of low risk individuals  $\lambda$ , i.e.  $\kappa \leq \kappa_1(\lambda)$ . Incentive compatibility still binds and the equilibrium contracts are similar to the ones without overconfidence. In case 2, the fraction of overconfident individuals is at an intermediate level, i.e.,  $\kappa_1(\lambda) < \kappa \leq \kappa_2(\lambda)$ . Incentive compatibility no longer binds and the equilibrium contracts are qualitatively different from the equilibrium without overconfidence. In case 3, the fraction of overconfident individuals is high, i.e.,  $\kappa > \kappa_2(\lambda)$ . Insurance prices are sufficiently high so that both low risk and overconfident types do not insure. We represent these three cases in Figure 2 (where the dotted line captures the idea that, by definition, the sum of  $\kappa$  and  $\lambda$  must be smaller than one).

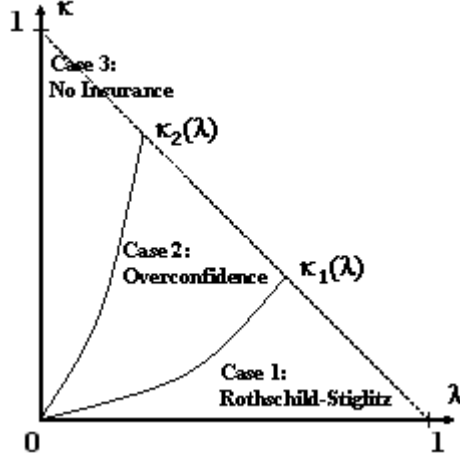


Figure 2: Equilibrium Diagram

**Case 1: “Rothschild-Stiglitz” Equilibrium** Assume that the fraction of overconfident agents  $\kappa$  is small relative to the fraction of low risk individuals  $\lambda$ , i.e.  $\kappa < \kappa_1(\lambda)$ . Then, the equilibrium contracts  $(\alpha^{LO}, \alpha^H)$  are shown in Figure 3. The only difference from the case without overconfidence is that the contract  $\alpha^{LO}$  must lie on the zero-profit line  $\pi_{LO} = 0$ , since it is chosen by low-risk and overconfident agents alike. The low risk and overconfident agents insurance coverage  $\alpha_1^{LO} + \alpha_2^{LO}$  increases as the fraction of overconfident individuals  $\kappa$  becomes larger. This follows because a counterclockwise rotation of the zero-profit line  $\pi_{LO} = 0$  makes the contract  $\alpha^{LO}$  shift in the direction of the 45 degree line (where insurance is full), along the high risk agents indifference curve  $I_H$ . An increment in  $\kappa$  also makes low risk agents worse off because it increases their insurance price  $P^{LO}$ . To see this note that as the zero-profit line  $\pi_{LO} = 0$  rotates towards  $\pi_H$ , its intersection with  $I_H$  moves below the low risk agents’ indifference curve  $I_L$ . Because the incentive compatibility constraint is binding, the overconfident and high-risk average equilibrium utilities coincide. They are not affected by changes in  $\kappa$ .

**Case 2. Pure-Overconfidence Equilibrium** When the fraction of overconfident individuals is intermediate, i.e.,  $\kappa_1(\lambda) < \kappa \leq \kappa_2(\kappa)$ , the equilibrium is represented in Figure 4. The incentive compatibility constraint no longer binds. To see this, let  $\bar{\alpha}$  be the intersection of the zero-profit line  $\pi_{LO} = 0$  with the indifference curve  $I_H$  passing through  $\alpha^H$ . Note that the indifference curve of low risk agents passing through  $\bar{\alpha}$  is steeper than the zero-profit line  $\pi_{LO} = 0$  (in contrast, in Figure 3 it was flatter). Hence,  $\bar{\alpha}$  is no longer an equilibrium because contracts

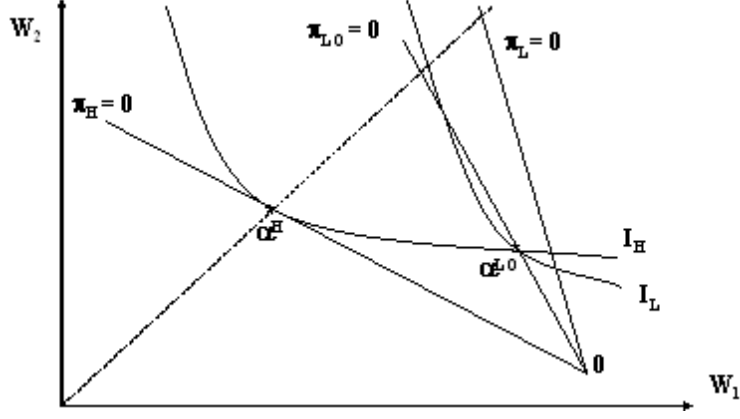


Figure 3: “Rothschild and Stiglitz” Equilibrium (Case 1).

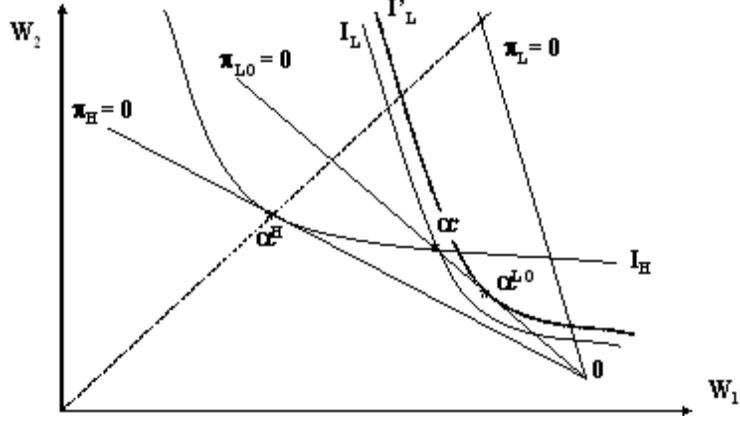


Figure 4: “Overconfidence” Equilibrium (Case 2).

such as  $\alpha^{LO}$  would make strictly positive profits.<sup>17</sup> The equilibrium contract for low risk and overconfident agents, denoted by  $\alpha^{LO}$ , is determined by the tangency point of the indifference curve  $I_L$  on the zero-profit line  $\pi_{LO} = 0$ . Under some regularity assumptions, the low risk and overconfident individuals’ equilibrium coverage  $\alpha_1^{LO} + \alpha_2^{LO}$  decreases in  $\kappa$ .<sup>18</sup> The low risk and overconfident individuals’ equilibrium utilities decrease in  $\kappa$ .

### Case 3. Overconfidence Equilibrium with uninsured Drivers

When the fraction of overconfident individuals is large,  $\kappa > \kappa_2(\kappa)$ , the incentive compatibility still does not bind. Furthermore, the zero-profit line  $\pi_{LO} = 0$  is sufficiently close to the zero-profit

<sup>17</sup>Low risk and overconfident agents prefer the contract  $\alpha^{LO}$  to the contract  $\bar{\alpha}$ . High-risk agents still prefer  $\alpha^H$  to the  $\alpha^{LO}$ .

<sup>18</sup>Specifically, this result holds if the coefficient of Relative Risk Aversion  $-wU''(w)/U'(w)$  is smaller than the bound  $(W - d)/W$  for any wealth amount  $w \in [W - d, W]$ .

line  $\pi_H = 0$  so that it becomes flatter than the indifference curve  $I_L$  that passes through the no-insurance contract 0. Hence, a corner solution  $\alpha^{LO} = 0$  is obtained. Low risk and overconfident agents believe that the insurance contracts they are offered are so unfavorable that they do not insure. This illustrates our first result.

**Result 1** *If the fraction  $\kappa$  of overconfident individuals is large enough relative to the fraction of low-risk individuals  $\lambda$ , i.e., if  $\kappa > \kappa_2(\lambda)$ , then low-risk and overconfident individuals choose to be uninsured in equilibrium.*

This result gives a simple account for the fact that a large fraction of the U.S. population has no health insurance. Indeed, an estimated 15.2 percent of the U.S. population were uninsured in 2002, according to the US Census.<sup>19</sup> Furthermore, according to the Insurance Research Council (IRC), an average of 14.9% of motorists were uninsured between 1989 and 1997. An alternative theoretical explanation is that severely budget-constrained motorists do not to insure and declare bankruptcy in case of accident where they are at fault (see Smith and Wright 1992). This explanation and our overconfidence hypothesis complement each other. In the Public Attitude Monitor (2000 and 2003) surveys, about 41% of the interviewed individuals reported reasons for non-insurance consistent with the limited-liability hypothesis and 31% report reasons consistent with overconfidence.<sup>20</sup>

## 5 Policy Recommendations

### 5.1 Can Compulsory Insurance be a Pareto Improvement?

**Compulsory Insurance without Overconfidence** A *compulsory insurance* requirement is a contract  $\beta = (\beta_1, \beta_2) > 0$ , that makes zero profits if imposed uniformly across all individuals. Each individual is required to buy contract  $\beta$  and is free to buy additional insurance  $\alpha(\beta)$  on top of  $\beta$ . Formally, let  $p_{LH} \equiv (1 - \lambda)p_H + \lambda p_L$  be the average probability of accident in the economy. Any compulsory insurance contract  $\beta$  that keeps the budget balanced must lie on the zero-profit line  $\pi_{LH} = 0$  i.e.,  $(1 - p_{LH})\beta_1 - p_{LH}\beta_2 = 0$ .

In adverse-selection insurance models without overconfidence, the introduction of compulsory

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<sup>19</sup>Young adults (18 to 24 years old) are less likely than any other population segment to be insured. Overconfidence is also particularly pervasive among young adults.

<sup>20</sup>The remaining 28% of interviewed individuals gave reasons consistent with both hypothesis.

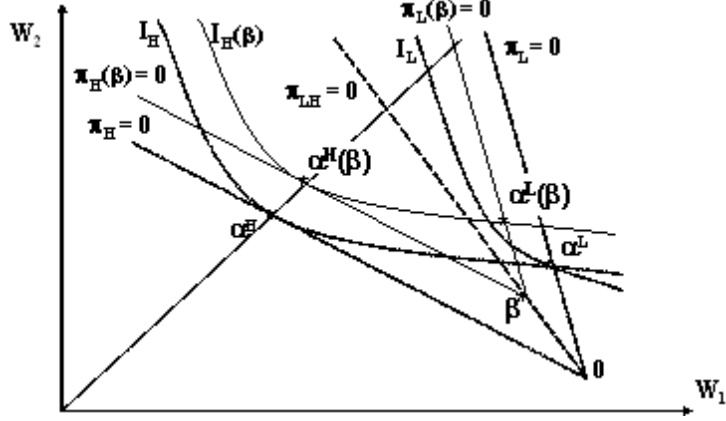


Figure 5: Compulsory Insurance without Overconfidence

insurance is a Pareto improvement, as long as the fraction of low-risk individuals is above a threshold (see Wilson, 1977, and Dahlby, 1981). To see this, recall that agents can buy insurance coverage in addition to the minimum required  $\beta$ . So, the adoption of  $\beta$  is equivalent to a change of endowment from  $(W, W - d)$  to  $(W - \beta_2, W - d + \beta_1)$ . Given this, the remainder of the analysis is qualitatively unchanged. High risk agents' contract  $\alpha^H(\beta)$  fully insure. Low risk agents contract  $\alpha^L(\beta)$  lies in the intersection of the zero-profit line  $\pi_L(\beta) = 0$  and the indifference curve  $I_H$  passing through  $\alpha^H(\beta)$  (see Figure 5).

The adoption of compulsory insurance makes high risk individuals better off, because the terms of the compulsory contract  $\beta$  are more favorable than the terms of the equilibrium contract  $\alpha^H$ . Low risk individuals are affected in two ways. On the one hand they pay the cost of being pooled together with the high risk individuals for the budget-balanced contract  $\beta$ . On the other hand, the adoption of compulsory insurance relaxes the incentive compatibility constraint imposed by the high risk subscribers. This can be seen in Figure 5. The introduction of the compulsory insurance contract  $\beta$  shifts the indifference curve  $I_H$  up. When the fraction of high risk subscribers is sufficiently small, the relaxation of incentive compatibility compensates low risk agents for the cost of subsidizing high risk individuals through  $\beta$ . This ultimately makes low risk agents better off.

**Compulsory Insurance and Overconfidence** Now consider the case in which there is a significant fraction of overconfident agents in the economy (i.e.,  $\kappa > \kappa_1(\lambda)$ , as in Figure 2).<sup>21</sup> Because the incentive-constraint is not binding in equilibrium, we show that the introduction of compulsory insurance cannot Pareto improve upon the *laissez-faire* equilibrium. It makes low risk individuals worse off. Compulsory insurance is equivalent to a transfer of wealth from low risk agents to high risk and overconfident agents without any beneficial effect on incentive-compatibility constraints.

**Result 2** *If the fraction of overconfidence agents in the economy is sufficiently large, i.e.  $\kappa > \kappa_1(\lambda)$ , then any compulsory insurance contract  $\beta > 0$  reduces the expected utility of low risk agents.*

This result may be appreciated by inspecting Figure 6. The low risk and overconfident individuals' zero-profit line  $\pi_{LO} = 0$  lies below the low risk individuals' indifference curve  $I_L$  passing through the equilibrium contract  $\alpha^{LO}$ . Any budget balanced compulsory insurance contract  $\beta$  lies on the zero-profit line  $\pi_{LH} = 0$ , which is strictly below the zero-profit line  $\pi_{LO} = 0$ . Hence, any contract  $\alpha^{LO}(\beta)$  purchased on top of a compulsory insurance contract  $\beta$  will also lie below the zero-profit line  $\pi_{LO} = 0$ , and therefore below the indifference curve  $I_L$ . Thus, low risk agents prefer the *laissez-faire* contract  $\alpha^{LO}$  over any allocation resulting from the introduction of compulsory insurance.

Remarkably, this result extends to any general mechanism: In contrast to the case without overconfidence, government intervention cannot Pareto improve upon the equilibrium outcome.

**Result 3** *If the fraction of overconfidence agents in the economy is sufficiently large, i.e. if  $\kappa > \kappa_1(\lambda)$ , then the competitive equilibrium cannot be Pareto improved by any incentive-compatible budget-balanced mechanism.*

This result is formally derived in appendix B which provides a mechanism design analysis. The basic intuition is as follows: Because the incentive-compatibility constraint is not binding, the equilibrium contract  $\alpha^{LO}$  gives the highest utility to low risk agents among all contracts on

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<sup>21</sup>If  $\kappa < \kappa_1(\lambda)$ , the analysis is analogous to the case without overconfidence.

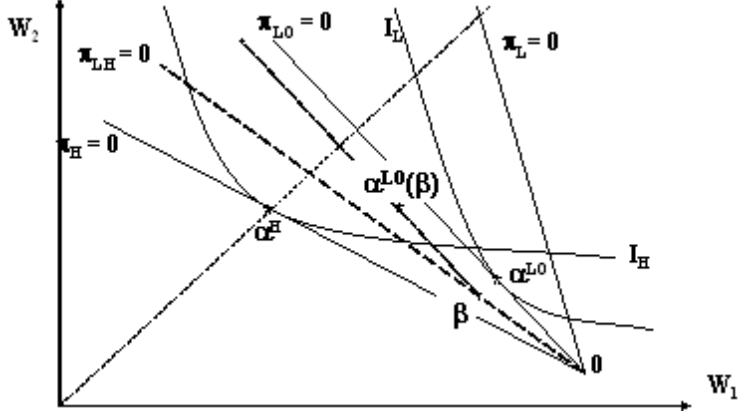


Figure 6: Compulsory Insurance with Overconfidence

the zero profit line  $\pi_{LO} = 0$  (see Figure 6). Budget-balanced government intervention leads to an outcome on the zero-profit condition  $\pi_{LO} = 0$ . This can only make low risk individuals worse off.

## 5.2 Training Programs

We now consider policies that reduce overconfidence in the economy. In the context of automobile insurance, such policies materialize in training programs that improve the drivers' self-assessment skills. We abstract from some effects that such training programs might yield, such as positive externalities involved in the improvement of road safety. Because overconfident agents' equilibrium perceived utility coincides with low-risk agents utility, we only report overconfident agents' actual utility. We start our analysis under the hypothesis that participation in such programs is mandatory, followed by a comparison to voluntary participation.

**Mandatory Self-assessment Training Programs** A *self-assessment training* program may succeed in changing overconfident agents' beliefs. At cost  $c > 0$ , each overconfident individual becomes aware of her high risk with probability  $\eta$ . Other agents' beliefs are not changed by the program. This leads to a reduction of the fraction  $\kappa$  of overconfident individuals in the economy. As seen in Section 4, this leads to a lower insurance price  $P^{LO}$  for low risk and overconfident subscribers.<sup>22</sup> This price reduction makes low risk individuals better off, as long as the cost  $c$

<sup>22</sup>Provided that  $\kappa < \kappa_2(\lambda)$  so that in equilibrium low risk and overconfident agents purchase strictly positive insurance. If  $\kappa > \kappa_2(\lambda)$  then low risk individuals are initially uninsured. To be beneficial, the mandatory training program need to reduce  $\kappa$  to the point where they start buying insurance again. This requires that  $\eta$  is sufficiently large.



is not too large. Low risk individuals do not have any direct benefit from the training program, because they cannot improve their already unbiased self-assessment. They only benefit indirectly through the reduction of the price of insurance  $P^{LO}$ . The equilibrium contract  $\alpha^H$  for high-risk individuals remains the same after the introduction of the training program. Hence they are worse off because of the training cost  $c$ . Whether overconfident individuals benefit or not depends on how large is their fraction  $\kappa$  before training. Following the analysis of Section 4, they benefit if and only if  $\kappa > \kappa_1(\lambda)$ . When  $\kappa > \kappa_1(\lambda)$ , their utility is decreasing in  $P^{LO}$  and smaller than the high-risk agents utility (see Figure 4). Hence they benefit both directly because they revise their beliefs with probability  $\eta$ , and indirectly through the reduction of insurance price  $P^{LO}$ . When  $\kappa < \kappa_1(\lambda)$ , the incentive compatibility binds (see Figure 3), and high-risk and overconfident individuals' actual utilities coincide. Hence also overconfident agents are worse off. This is summarized below.

**Result 4** *Low risk agents benefit from a compulsory self-assessment training program, as long as benefits  $\eta$  are sufficiently large and costs  $c$  are sufficiently low. Overconfident individuals are better off if and only if  $\kappa > \kappa_1(\lambda)$ . High risk individuals are worse off.*

The failure of mandatory training programs to be a Pareto improvement motivates us to consider voluntary training programs.

**Voluntary Self-assessment Training Programs** Suppose that participation to the training program is voluntary. If the training cost  $c$  is sufficiently small, then the equilibrium is as follows. Insurance contracts terms depend on attendance to the training program. The contracts  $\alpha^{LO}$  and  $\alpha^H$  are offered to agents who do not attend the program. These contracts are identical to those derived in section 4. In addition, agents who do attend the program are offered a better priced contract  $\hat{\alpha}^{LO}$ . The contract  $\hat{\alpha}^{LO}$  is intended for low-risk agents and for those agents who remain overconfident despite participating in the training program. All low risk individuals and overconfident agents join the training program, high-risk agents do not.

To see that this is the (unique) equilibrium, note that if the training cost  $c$  is sufficiently small, all the low risk and overconfident agents will be attracted to lower insurance prices and will join the training program.<sup>23</sup> Because all low-risk and overconfident agents join the program,

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<sup>23</sup>At the time they choose to join the training program, none of these agents believes that she will directly benefit

the fraction of overconfident individuals  $\kappa$  decreases, and this results in lower insurance prices.

As in the case of compulsory programs, low risk individuals benefit from the introduction of the voluntary training program, and overconfident individuals are better off if and only if their fraction is sufficiently large in the economy. High risk agents do not join the program and are not affected by it.

**Result 5** *If overconfidence is sufficiently common in the economy, i.e. if  $\kappa > \kappa_1(\lambda)$ , then the introduction of voluntary training programs is a Pareto improvement, as long as benefits  $\eta$  are sufficiently large and costs  $c$  are sufficiently low.*

**Practical Implementation of Training Programs** In the context of driving safety, a recent report published by Hatakka et al. (2002) for the European Union describes several techniques to reduce drivers' overconfidence. These techniques have been successfully tested on novice learners, experienced and professional drivers. Self-evaluative techniques are offered in France (Billard 1995). A reduction of traffic violations after participation in courses based on such techniques has been documented in Germany by Utzelmann and Jacobshagen (1997). Furthermore, Regan et al. (1998) found that overconfidence can be reduced through computer simulated driving runs. These findings are confirmed by Gregersen (1996) in an experiment sponsored by the Swedish National Road Administration, on a driving course that simulates icy-road conditions. Further references are available upon request.

## 6 Positive Implications of Overconfidence

**Risk aversion and Overconfidence** In models with one single decision maker, over-optimistic beliefs cannot be distinguished from a reduction of risk aversion. They both lead to greater willingness to take risks. However, they yield different implications in our model. We have shown in section 4 that adding a small fraction of overconfident individuals to a basic adverse selection model increases equilibrium insurance coverage (see Figure 3). In contrast, Figure 7 shows that adding a small fraction of high-risk individuals with reduced risk aversion reduces equilibrium insurance coverage.

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from the program. They all believe that their risk is low. They do not join to improve their self-assessment skill. They join only because  $\hat{\alpha}^{LO}$  is cheaper than the contract  $\alpha^{LO}$  that they would be offered without the program.

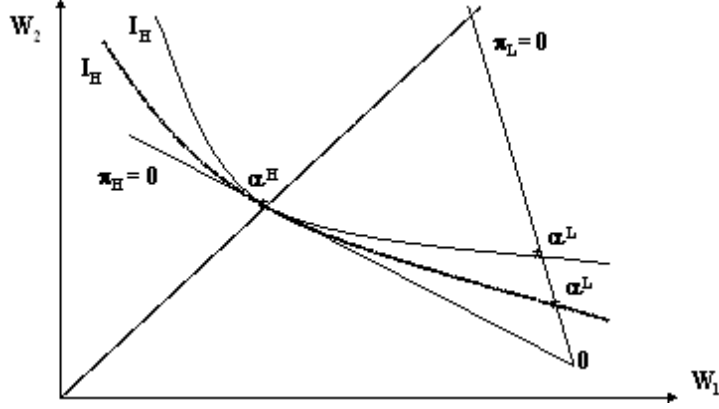


Figure 7: Adverse Selection with less risk-averse high-risk individuals

Risk neutral individuals are aware of their risk preference, whereas overconfident agents do not know they are overconfident. They believe incorrectly that their risk is low. This distinction is not quite significant in a decision theoretic model, but it is important in incomplete information games. Agents may signal different levels of risk aversion by purchasing different contracts, but overconfident agents cannot be separated in this way from unbiased agents with identical perceived risk.

**Risk Classification** In this section, we introduce risk classification. Each individual has a public signal  $x$ : her *risk class*. Individuals in different risk classes  $x$  are offered different contracts  $\alpha^{LO}(x)$  and  $\alpha^H(x)$ . We assume that the higher the risk class  $x$ , the more likely the individual's risk is high and the more likely that she is overconfident.<sup>24</sup> Hence,  $\kappa(x)$  (the fraction of overconfident agents in class  $x$ ) increases in  $x$ , whereas  $\lambda(x)$  (the fraction of low risk agents in class  $x$ ) decreases in  $x$ .

In the equilibrium of our model, low risk and overconfident agents choose the same contract  $\alpha^{LO}(x)$ . Its price  $P^{LO}(x) = p_{LO}(x) = [\kappa(x)p_H + \lambda(x)p_L]/[\kappa(x) + \lambda(x)]$  depends on the composition of overconfident and low-risk individuals which, in turn, is different in different risk classes. High risk agents in all risk classes still purchase the same contract  $\alpha^H$ . The insurance price  $P^H$  for high-risk agents is higher than  $P^{LO}(x)$  for every  $x$ . Hence, our model account for the following two stylized facts.

<sup>24</sup>In the context of automobile insurance, this relates to the experimental evidence that young male novice drivers are more likely to be overconfident than older and female drivers.

First, for any given coverage amount, insurance contract prices (of low-risk and overconfident agents) are higher for agents who belong to riskier classes. This is difficult to explain in insurance models with asymmetric information. In the equilibrium of the Rothschild and Stiglitz (1976) model, insurance firms do not make their contracts depend on risk classes. Regardless of the signal  $x$ , high-risk agents choose contract  $\alpha^H$ , and low-risk ones choose  $\alpha^L$ . Hence insurance prices  $P^L(x)$  and  $P^H(x)$  are independent of the signal  $x$ . In contrast, in our model the price  $P^{LO}(x)$  of contracts offered to low-risk and overconfident agents increases in  $x$ .<sup>25</sup>

Second, the data show a large heterogeneity in insurance price that is not justified by risk classes (e.g. Chiappori and Salanié, 2001, page 63). This fact is difficult to explain in models with symmetric information. If the risk class is a sufficient statistics of each agent's actual risk, then perfect competition implies that the equilibrium price  $P(x)$  of an agent's insurance contract  $\alpha(x)$  depends only on her risk class  $x$ . Instead, in our model the insurance price  $P^H$  for high-risk agents is higher than  $P^{LO}(x)$  within each risk class  $x$ .

**Insurance Coverage** The recent empirical studies that we discuss in the introduction study the relationship between insurance coverage and ex-post risk, conditional on the risk class, to investigate for the presence of asymmetric information in insurance markets. Models with asymmetric information predict a positive relation, while the relationship is indeterminate under symmetric information. We now show that a simple extension of our model of insurance markets with overconfidence may deliver a non-monotonic relationship between ex-post risk and equilibrium coverage.<sup>26</sup>

We extend our basic model by supposing that the overconfident agents' risk  $p_O$  is higher than the high risk agents' risk  $p_H$ , i.e., we assume that  $p_O > p_H$ . In a wide parameter range, the equilibrium coverage of high risk agents is higher than the equilibrium coverage of low risk and overconfident agents (because high risk agents fully insure). Conditional on the risk class  $x$ , the average risk of low risk and overconfident agents is  $p_{OL}(x) \equiv [\kappa(x)p_O + \lambda(x)p_L]/[\kappa(x) + \lambda(x)]$ .

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<sup>25</sup>In the context of automobile insurance, this relates to experimental evidence that young male novice drivers are more likely overconfident than older and than female ones. Also, a bad driving record may be understood as evidence of overconfident, reckless driving habits. The insurance firms learn that the driver is likely to be risky, but the driver maintains her overconfident belief of being a first-class driver.

<sup>26</sup>The relevance of overconfidence for the relationship between risk and coverage has also been independently singled out by Koufopoulos (2003). Unlike our model, his model does not allow for heterogeneity in actual risk conditional on perceived risk, thus precluding our policy results.

It follows that the relationship between insurance coverage and risk is positive if  $p_{OL}(x) \leq p_H$ . On the other hand, if the fraction of overconfident individuals is sufficiently large (relative to the fraction of low risk agents) then  $p_{OL}(x) > p_H$  and, therefore, the relationship between insurance coverage and risk becomes negative. Given that  $p_{OL}(x)$  is increasing in  $x$ , our model may deliver a positive relationship between equilibrium coverage and ex-post risk among agents in lower risk classes and a negative relationship between equilibrium coverage and ex-post risk among agents in higher risk classes.<sup>27</sup>

While the relationship between actual risk and coverage is non-monotonic, the relationship between insurance coverage and self-reported risk, conditional on risk classification, is positive. In fact, the perceived risk of both overconfident and low risk agents is smaller than the risk of high risk agents (and the equilibrium coverage of high risk agents is higher than the equilibrium coverage of overconfident and low risk agents). This distinguishes our model from models of insurance with symmetric information, where this relationship is indeterminate.

## 7 Conclusion

Compulsory insurance leads to a Pareto improvement in adverse selection models that abstract from overconfidence. On the other hand, if there is a significant fraction of overconfident agents in the economy, then compulsory insurance (and any other government intervention) does not Pareto improve the equilibrium, because it is detrimental to low risk individuals. Instead, voluntary training programs that improve self-assessment skills make all agents better off, as long as the training costs are sufficiently low.

Introducing overconfidence in standard insurance markets with adverse selection also provides novel positive implications such as prices heterogeneity both within and across risk classes and an indeterminate relationship between insurance coverage and ex-post risk. In contrast, a positive relationship between insurance coverage and self-reported risk is robust to the addition of overconfident agents.

Our model delivers a simple demonstration that behavioral biases need not be the basis for

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<sup>27</sup>Consistently with this prediction, Chiappori and Salanié (2001) find a (non statistically significant) negative relation between unobservable risk and coverage for young drivers, whereas Cohen (2003) finds a significant and positive relation for mature drivers.

the imposition of paternalistic policies in the context of an insurance model. We hope that a proper combination of the literatures of behavioral economics and mechanism design will clarify the implications of behavioral biases for policy making in a variety of contexts such as savings programs, investors' protection, and education planning.

## A Appendix: Formal Analysis

**Definition of Locally-Competitive Equilibrium.** A *locally-competitive equilibrium* is a set of contracts  $A$  such that when each contract  $\alpha \in A$  is available in the market, (i) no contract  $\alpha \in A$  makes negative expected profits, and (ii) there is an  $\varepsilon > 0$  such that any contract  $\alpha'$  for which  $\|\alpha - \alpha'\| < \varepsilon$  for any  $\alpha \in A$ , would not make strictly positive profits.

This equilibrium may be interpreted as the stable outcome of a process, where insurance firms are only willing to introduce small deviations from the insurance contracts already offered in the market.<sup>28</sup> Clearly, any perfectly-competitive equilibrium is also locally-competitive, but not viceversa.

**Equilibrium Analysis.** This section formalizes the graphical equilibrium analysis of Section 4, and proves Result 1.

The equilibrium analysis proceeds in two steps. The first one hinges on the insight that insurance firms cannot screen between overconfident and low-risk individuals, because their beliefs are the same.

**Proposition A.1** *In the unique locally-competitive equilibrium, high-risk individuals choose the contract  $\alpha^H = (p_H d, (1 - p_H)d)$ . Low-risk and overconfident individuals choose the contract  $\alpha^{LO}$  that solves the maximization problem*

$$\max_{\alpha} V(W, d; p_L, \alpha), \tag{A.1}$$

*subject to the non-negativity constraint  $\alpha \geq 0$ , and to the following Incentive-Compatibility and Zero-Profit conditions:*

$$V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha), \tag{A.2}$$

$$(1 - p_{LO}) \alpha_1 - p_{LO} \alpha_2 = 0. \tag{A.3}$$

*As long as  $\alpha^{LO} > 0$ , the insurance price  $P^{LO}$  equals  $p_{LO}$  and increases in  $\kappa$ .*

**Proof.** The proof that the only possible competitive equilibrium is a separating equilibrium where type  $H$  individuals purchase contract  $\alpha^H = (p_H d, (1 - p_H)d)$  is analogous to the analysis

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<sup>28</sup>Riley (1979) shows that the locally-competitive equilibrium coincides with a “reactive” equilibrium concept where each firm, before introducing new contracts, anticipates that firms already in the market will react by offering new contracts, as long as they generate positive profits. See Wilson (1977) for an alternative “reactive” equilibrium. In a general-equilibrium model, Dubey and Geanakoplos (2002) establish the general existence of a separating equilibrium that approximates the locally-competitive equilibrium.

of Rothschild and Stiglitz (1976). Suppose by contradiction that in equilibrium, type  $L$  and  $O$  individuals buy distinct contracts  $\alpha'$  and  $\alpha''$  with probabilities  $(\sigma'_O, \sigma'_L)$  and probabilities  $(\sigma''_O, \sigma''_L)$ ; it must be that  $V(W, d; p_L, \alpha') = V(W, d; p_L, \alpha'')$  or else it cannot be that both  $\alpha'$  and  $\alpha''$  are purchased in equilibrium. Introduce the average damage probabilities

$$p'_{LO} = \frac{\kappa\sigma'_O p_H + \lambda\sigma'_L p_L}{\kappa\sigma'_O + \lambda\sigma'_L}, \quad p''_{LO} = \frac{\kappa\sigma''_O p_H + \lambda\sigma''_L p_L}{\kappa\sigma''_O + \lambda\sigma''_L},$$

perfect competition requires that  $p'_{LO}\alpha'_2 = (1 - p'_{LO})\alpha'_1$  and  $p''_{LO}\alpha''_2 = (1 - p''_{LO})\alpha''_1$ ; if  $p'_{LO} = p''_{LO}$ , then  $\alpha' = \alpha''$  and we have a contradiction. Say that  $p''_{LO} > p'_{LO}$ , then it must be that  $p''_{LO} > p_{LO}$ . Since  $U$  is twice differentiable, it follows that there is an  $\varepsilon > 0$  small enough such that insurance firms can make strictly positive profits by selling a contract  $\hat{\alpha} = (\alpha'_1 + (1 - \hat{p})\varepsilon, \alpha'_2 + \hat{p}\varepsilon)$  with  $\hat{p} \in (p_{LO}, p''_{LO})$ . This contract yields  $V(W, d; p_L, \hat{\alpha}) > V(W, d; p_L, \alpha'')$ . Hence it is preferred by all type  $L$  and  $O$  agents to contracts  $\alpha'$  and  $\alpha''$ , but not by type  $H$  agents.

Because  $p_H > p_L$ ,  $dp_{LO}/d\lambda < 0$  and  $dp_{LO}/d\kappa > 0$ . The price  $P^{LO} = \alpha_1^{LO}/(\alpha_1^{LO} + \alpha_2^{LO})$  equals  $p_{LO}$  by substituting in the zero-profit condition (A.3). Hence it increases in  $\kappa$ . ■

The complete equilibrium characterization follows from the insight that the incentive-compatibility condition need not be binding in equilibrium. The characterization is summarized in the Proposition A.2 below. In particular, we prove Result 1. Proposition A.2 also reports our comparative statics results.

For any fixed parameter constellation  $(W, d, p_H, p_L)$ , the thresholds  $\kappa_1$  and  $\kappa_2$  presented in Figure 2, as functions of  $\lambda$ , uniquely solve respectively:

$$V(W, d; p_H, \alpha) = U(W - p_H d), \quad p_{LO}\alpha_2 = (1 - p_{LO})\alpha_1, \quad \frac{(1 - p_L)U'(W - \alpha_1)}{p_L U'(W - d + \alpha_2)} = \frac{1 - p_{LO}}{p_{LO}}; \quad (\text{A.4})$$

$$\frac{(1 - p_L)U'(W)}{p_L U'(W - d)} = \frac{1 - p_{LO}}{p_{LO}}. \quad (\text{A.5})$$

where the variables  $\kappa$  and  $\lambda$  are embedded in the expression  $p_{LO} = [\kappa p_L + \lambda p_H]/[\kappa + \lambda]$ .

**Proposition A.2** *For  $\kappa < \kappa_1(\lambda)$ , the Incentive-Compatibility condition (A.2) binds at the equilibrium contract  $\alpha^{LO}$ . The insurance coverage  $\alpha_1^{LO} + \alpha_2^{LO}$  increases in  $\kappa$ , the low risk agents' utility  $V(W, d; p_L, \alpha^{LO})$  decreases in  $\kappa$ , the overconfident individuals' utility  $V(W, d; p_H, \alpha^{LO})$  equals the high-risk individuals' utility  $V(W, d; p_L, \alpha^H)$ , they are constant in  $\kappa$ . For  $\kappa_1(\lambda) < \kappa < \kappa_2(\lambda)$ , the equilibrium contract  $\alpha^{LO}$  satisfies the tangency condition*

$$\frac{(1 - p_L)U'(W - \alpha_1^{LO})}{p_L U'(W - d + \alpha_2^{LO})} = \frac{1 - p_{LO}}{p_{LO}}. \quad (\text{A.6})$$

Hence  $V(W, d; p_H, \alpha^{LO}) < V(W, d; p_H, \alpha^H)$ , and both  $V(W, d; p_L, \alpha^{LO})$  and  $V(W, d; p_H, \alpha^{LO})$  decrease in  $\kappa$ , as long as the Arrow-Pratt coefficient of Relative Risk Aversion of  $U$  is bounded by  $(W - d)/d$ . For  $\kappa > \kappa_2(\lambda)$ , low-risk and overconfident individuals are uninsured in equilibrium:  $\alpha^{LO} = 0$ .

**Proof.** Let  $\bar{\alpha} = (\bar{\alpha}_1, \bar{\alpha}_2)$  be the contract pinned down by the zero-profit condition (A.3) and by the binding incentive-compatibility condition (A.2). Differentiating these last two equations, we obtain:

$$(1 - p_{LO})d\bar{\alpha}_1 - p_{LO}d\bar{\alpha}_2 - \bar{\alpha}_1 dp_{LO} - \bar{\alpha}_2 dp_{LO} = 0 \quad (\text{A.7})$$

$$p_H U'(W - d + \bar{\alpha}_2)d\bar{\alpha}_2 = (1 - p_H)U'(W - \bar{\alpha}_1)d\bar{\alpha}_1, \quad (\text{A.8})$$

$$\frac{d\bar{\alpha}_1}{dp_{LO}} = \frac{(\bar{\alpha}_1 + \bar{\alpha}_2)p_H U'(W - d + \bar{\alpha}_2)}{\Delta} > 0, \quad \frac{d\bar{\alpha}_2}{dp_{LO}} = \frac{(\bar{\alpha}_1 + \bar{\alpha}_2)(1 - p_H)U'(W - \bar{\alpha}_1)}{\Delta} > 0, \quad (\text{A.9})$$

where the quantity  $\Delta = (1 - p_{LO})p_H U'(W - d + \bar{\alpha}_2) - p_{LO}(1 - p_H)U'(W - \bar{\alpha}_1)$  is positive because  $U'' < 0$ ,  $-\bar{\alpha}_1 > -d + \bar{\alpha}_2$  and  $p_H > p_{LO}$ . Because  $dp_{LO}/d\lambda < 0$  and  $dp_{LO}/d\kappa > 0$ , we obtain that  $d\bar{\alpha}_1/d\kappa > 0$ ,  $d\bar{\alpha}_1/d\lambda < 0$ ,  $d\bar{\alpha}_2/d\kappa > 0$ , and  $d\bar{\alpha}_2/d\lambda < 0$ .

Let  $Q = \frac{(1-p_L)U'(W-\bar{\alpha}_1)}{p_L U'(W-d+\bar{\alpha}_2)}$  and  $q = \frac{1-p_{LO}}{p_{LO}}$ . Differentiating  $Q$ , we obtain

$$dQ = \frac{1 - p_L}{p_L} \left[ -\frac{U''(W - \bar{\alpha}_1)}{U'(W - d + \bar{\alpha}_2)} d\bar{\alpha}_1 - \frac{U''(W - d + \bar{\alpha}_2)U'(W - \bar{\alpha}_1)}{(U'(W - d + \bar{\alpha}_2))^2} d\bar{\alpha}_2 \right].$$

Hence  $dQ/d\kappa > 0$ . Because  $dq/dp_{LO} < 0$  and  $dp_{LO}/d\kappa > 0$ , we have shown that for any  $\lambda$ , there is a unique threshold  $\kappa_1$  pinned down by system (A.4) and that

$$\frac{(1 - p_L)U'(W - \bar{\alpha}_1)}{p_L U'(W - d + \bar{\alpha}_2)} > (<) \frac{1 - p_{LO}}{p_{LO}} \text{ if and only if } \kappa > (<) \kappa_1(\lambda).$$

Because  $dQ/d\lambda < 0$ ,  $dq/dp_{LO} < 0$  and  $dp_{LO}/d\lambda < 0$ ,  $\kappa_1$  is strictly increasing in  $\lambda$  by the Implicit Function Theorem.

Suppose that  $\kappa < \kappa_1(\lambda)$ , and suppose by contradiction that in the locally-competitive equilibrium  $\alpha^{LO}$ , the Incentive-Compatibility condition (A.2) is not binding:  $(1 - p_H)U(W - \alpha_1^{LO}) + p_H U(W - d + \alpha_2^{LO}) < U(W - p_H d)$ . Since  $U'' < 0$ , and both  $\bar{\alpha}$  and  $\alpha^{LO}$  satisfy the Zero-Profit condition (A.3), it must be the case that  $\bar{\alpha} < \alpha^{LO}$  and hence that:

$$\frac{(1 - p_L)U'(W - \alpha_1^{LO})}{p_L U'(W - d + \alpha_2^{LO})} < \frac{(1 - p_L)U'(W - \bar{\alpha}_1)}{p_L U'(W - d + \bar{\alpha}_2)} < \frac{1 - p_{LO}}{p_{LO}}.$$

Since  $U$  is twice differentiable, it follows that there is an  $\varepsilon > 0$  small enough such that for any  $\beta \in \left( \frac{(1-p_L)U'(W-\alpha_1^{LO})}{p_L U'(W-d+\alpha_2^{LO})}, \frac{1-p_{LO}}{p_{LO}} \right)$  the insurance contract  $\alpha^{LO} + \varepsilon(1, \beta)$  is chosen by type  $L$  and  $O$  but not by type  $H$ , and makes strictly positive profit. This concludes that for  $\kappa < \kappa_1(\lambda)$ ,  $\alpha^{LO} = \bar{\alpha}$ .

The result that  $d(\alpha_1^{LO} + \alpha_2^{LO})/d\kappa > 0$  follows from the inequalities (A.9). The expected utility  $V(W, d; p_L, \alpha^{LO})$  decreases in  $\kappa$  and increases in  $\lambda$  because  $dp_{LO}/d\kappa > 0$ ,  $dp_{LO}/d\lambda < 0$  and

$$\begin{aligned} \frac{\partial}{\partial p_{LO}} V(W, d; p_L, \alpha^{LO}) &= -(1 - p_L)U'(W - \alpha_1^{LO}) \frac{\partial \alpha_1^{LO}}{\partial p_{LO}} + p_L U'(W - d + \alpha_2^{LO}) \frac{\partial \alpha_2^{LO}}{\partial p_{LO}} \\ &= -(1 - p_L)U'(W - \alpha_1^{LO}) \frac{\alpha_1^{LO} + \alpha_2^{LO}}{1 - p_{LO}} - p_L U'(W - d + \alpha_2^{LO}) \frac{\alpha_1^{LO} + \alpha_2^{LO}}{p_{LO}} < 0, \end{aligned}$$



by substituting in condition (A.7). Because the Incentive Compatibility condition (A.2) binds, we obtain that  $V(W, d; p_H, \alpha^{LO}) = V(W, d; p_L, \alpha^H)$ . Both quantities are constant in  $\kappa$ .

Suppose that  $\kappa > \kappa_1(\lambda)$ , and hence that

$$\frac{(1-p_L)U'(W-\bar{\alpha}_1)}{p_L U'(W-d+\bar{\alpha}_2)} > \frac{1-p_{LO}}{p_{LO}}.$$

Suppose by contradiction that  $\alpha^{LO} = \bar{\alpha}$  in equilibrium. Since  $U'' < 0$  and  $U$  is smooth, for any  $\varepsilon > 0$  small enough, and  $\beta \in \left( \max \left\{ \frac{1-p_{LO}}{p_{LO}}, \frac{(1-p_H)U'(W-\bar{\alpha}_1)}{p_H U'(W-d-\bar{\alpha}_2)} \right\}, \frac{(1-p_L)U'(W-\bar{\alpha}_1)}{p_L U'(W-d+\bar{\alpha}_2)} \right)$ , the contract  $\bar{\alpha} - \varepsilon(1, \beta)$  is chosen only by types  $L$  and  $O$ , and not by type  $H$ , and yields strictly positive profit.

Because the Incentive Compatibility condition (A.2) does not bind,  $\alpha^{LO}$  solves the problem

$$\max_{\alpha} V(W, d; p_L, \alpha) \quad \text{s.t.} \quad p_{LO}\alpha_2 = (1-p_{LO})\alpha_1, \quad \alpha \geq 0. \quad (\text{A.10})$$

Since  $dp_{LO}/d\kappa > 0$ , for any  $\lambda$  there is a unique threshold  $\kappa_2(\lambda)$  such that:

$$\frac{(1-p_L)U'(W)}{p_L U'(W-d)} > (<) \frac{1-p_{LO}}{p_{LO}} \text{ if and only if } \kappa > (<) \kappa_2(\lambda).$$

When  $\kappa > \kappa_2(\lambda)$ , the constraint  $\alpha \geq 0$  binds in the maximization problem (A.10), whereas when  $\kappa_1(\lambda) < \kappa < \kappa_2(\lambda)$ , the equilibrium contract  $\alpha^{LO}$  is pinned down by the zero-profit condition (A.3) and the tangency condition (A.6). Since  $dp_{LO}/d\lambda < 0$ , the threshold function  $\kappa_2$  is increasing in  $\lambda$ .

To study the effect of  $\kappa$  on  $\alpha_1^{LO} + \alpha_2^{LO}$ , we differentiate the zero-profit condition (A.3) and the tangency condition (A.6) with respect to the quantity  $q$ , decreasing in  $p_{LO}$  and hence in  $\kappa$ ,

$$\begin{aligned} \frac{\partial \alpha_2^{LO}}{\partial q} &= \alpha_1^{LO} + q \frac{\partial \alpha_1^{LO}}{\partial q}, \\ - (1-p_L)U''(W-\alpha_1^{LO}) \frac{\partial \alpha_1^{LO}}{\partial q} &= p_L U'(W-d+\alpha_2^{LO}) + q p_L U''(W-d+\alpha_2^{LO}) \frac{\partial \alpha_2^{LO}}{\partial q} \end{aligned}$$

rearranging we obtain:

$$\frac{\partial (\alpha_1^{LO} + \alpha_2^{LO})}{\partial q} = \alpha_1^{LO} - (1+q)p_L \frac{U'(W-d+\alpha_2^{LO}) + U''(W-d+\alpha_2^{LO})\alpha_2^{LO}}{(1-p_L)U''(W-\alpha_1^{LO}) + q^2 p_L U''(W-d+\alpha_2^{LO})}.$$

This fraction is positive if the numerator is positive, which follows from the assumption that  $-U'(w)w/U''(w) < (W-d)/d$ .

Low-risk individuals utility  $V(W, d; p_L, \alpha^{LO})$  decreases in  $p_{LO}$  –hence decreasing in  $\kappa$  and increasing in  $\lambda$ – by a simple revealed-preference argument on the maximization problem (A.10). Hence, the overconfident agents' utility  $V(W, d; p_H, \alpha^{LO})$  decreases in  $p_{LO}$  as long as the insurance coverage  $\alpha_1^{LO} + \alpha_2^{LO}$  decreases in  $p_{LO}$ , because the Marginal Rate of Substitution  $\frac{(1-p_H)U'(W-\alpha_1^{LO})}{p_H U'(W-d+\alpha_2^{LO})}$  is larger than  $\frac{(1-p_L)U'(W-\alpha_1^{LO})}{p_L U'(W-d+\alpha_2^{LO})}$ . ■

We conclude by determining conditions for perfect-competitive equilibrium existence. For any fixed parameter constellation  $(W, d, p_H, p_L)$ , the threshold  $\lambda_0$  reported in Section 4, as function

of  $\kappa$ , solves the system:

$$\begin{cases} V(W, d; p_H, \alpha) = U(W - p_H d), & p_{LO}\alpha_2 = (1 - p_{LO})\alpha_1, & V(W, d; p_L, \beta) = V(W, d; p_L, \alpha) \\ p_{LH}\beta_2 = (1 - p_{LH})\beta_1, & \frac{U'(W - \beta_1)}{U'(W - d + \beta_2)} = \frac{[1 - p_{LH}]/p_{LH}}{[1 - p_L]/p_L}. \end{cases} \quad (\text{A.11})$$

**Proposition A.3** *The locally-competitive equilibrium  $(\alpha^H, \alpha^{LO})$  is also perfectly competitive if and only if  $\lambda < \lambda_0(\kappa)$ , where  $\lambda_0^{-1} < \kappa_1$ .*

**Proof.** The locally-competitive equilibrium  $(\alpha^{LO}, \alpha^H)$  is also the perfectly-competitive equilibrium contract if  $V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \beta)$ , where

$$\beta = \arg \max_{\alpha} V(W, d; p_L, \alpha) \quad \text{s.t.} \quad p_{LH}\alpha_2 \leq (1 - p_{LH})\alpha_1, \quad \alpha \geq 0. \quad (\text{A.12})$$

Hence pinning down condition A.11. When  $\kappa \geq \kappa_1(\lambda)$ , by construction,  $\alpha^{LO}$  solves the problem (A.10). Thus, by revealed preferences,  $V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \beta)$  because  $p_{LH} \geq p_{LO}$ , hence  $\alpha^{LO}$  is the perfectly-competitive equilibrium. Because  $V(W, d; p_L, \alpha^{LO})$  decreases in  $\kappa$  and increases in  $\lambda$ , whereas  $V(W, d; p_L, \beta)$  increases in  $\lambda$  but it is constant in  $\kappa$  ( $p_{LH}$  depends only on  $\lambda$ ), there is a unique strictly-increasing threshold  $\lambda_0^{-1} < \kappa_1$ , such that  $\alpha^{LO}$  is a perfectly-competitive equilibrium if and only if  $\kappa > \lambda_0^{-1}(\lambda)$ . ■

**Policy Recommendations** This section restates and proves our policy results, with the exception of Result 3 which is proved in Appendix B.

**Result 2** *If the fraction of overconfidence agents in the economy is sufficiently large, i.e.  $\kappa > \kappa_1(\lambda)$ , then any compulsory insurance contract  $\beta > 0$  reduces the expected utility of low risk agents.*

**Proof.** When  $\kappa > \kappa_1(\lambda)$ , Proposition A.2 concluded that the equilibrium contract  $\alpha^{LO}$  maximizes low-risk individuals' utility  $V(W, d; p_L, \alpha)$  among the allocations  $\alpha \geq 0$  such that  $p_{LO}\alpha_2 \leq (1 - p_{LO})\alpha_1$ . Any compulsory insurance  $\beta > 0$  is such that  $\beta_2/\beta_1 = \frac{1 - p_{LH}}{p_{LH}} < \frac{1 - p_{LO}}{p_{LO}}$ . Given that  $\beta > 0$  is adopted, the equilibrium contract  $\alpha^{LO}(\beta)$  of low-risk individuals is such that  $p_{LO}\alpha_2^{LO}(\beta) = (1 - p_{LO})\alpha_1^{LO}(\beta)$ , by Proposition A.1. Thus  $p_{LO}(\beta_2 + \alpha_2^{LO}(\beta)) < (1 - p_{LO})(\beta_1 + \alpha_1^{LO}(\beta))$ . By revealed preferences, the allocation  $\beta + \alpha^{LO}(\beta)$  yields smaller expected utility to low-risk individuals than the allocation  $\alpha^{LO}$ . ■

**Result 4** *Low risk agents benefit from a compulsory self-assessment training program, as long as benefits  $\eta$  are sufficiently large and costs  $c$  are sufficiently low. Overconfident individuals are better off if and only if  $\kappa > \kappa_1(\lambda)$ . High risk individuals are worse off.*

**Proof.** For any  $\kappa$  fraction of overconfident agents, let  $\alpha^{LO}(\kappa)$  be the associated equilibrium contract as calculated in Proposition A.2. When the training program is adopted, the fraction of overconfident agents changes from  $\kappa$  to  $\kappa' = (1 - \eta)\kappa < \kappa$ , and  $\lambda$  is unchanged. High-risk agents

are worse off by compulsory training because their equilibrium utility  $V(W, d; p_H, \alpha^H)$  is constant in  $\kappa$  and  $c > 0$ .

For any  $\eta$  large enough,  $\kappa' = (1 - \eta)\kappa < \kappa_2(\lambda)$ . By Proposition A.2, the low-risk equilibrium utility  $V(W, d, p_L, \alpha^{LO}(\kappa'))$  decreases in  $\kappa'$ , when  $\kappa' < \kappa_2(\lambda)$  and it is constant in  $\kappa'$  when  $\kappa' > \kappa_2(\lambda)$ . Hence, for  $c$  small enough,  $V(W - c, d, p_L, \alpha^{LO}(\kappa')) > V(W, d, p_L, \alpha^{LO}(\kappa))$ : low-risk agents benefit by compulsory training.

By Proposition A.2, when  $\kappa \geq \kappa_2(\lambda)$ , the equilibrium overconfident agents' utility  $V(W, d; p_H, \alpha^{LO})$  is constant in  $\kappa$ . When  $\kappa \in [\kappa_1(\lambda), \kappa_2(\lambda)]$ ,  $V(W, d; p_H, \alpha^{LO})$  decreases in  $\kappa$ . For any  $\kappa > \kappa_1$ ,  $V(W, d; p_H, \alpha^{LO})$  is smaller than the high-risk agents utility  $V(W, d, p_H, \alpha^H)$ . When  $\kappa < \kappa_1(\lambda)$ ,  $V(W, d, p_H, \alpha^H) = V(W, d; p_H, \alpha^{LO})$ , and both quantities are constant in  $\kappa$ . With probability  $\eta$  each overconfident type turns into a high-risk type, and with probability  $(1 - \eta)$  she remains overconfident. Hence she is better off with compulsory training if and only if  $\kappa > \kappa_1(\lambda)$ . ■

**Result 5** *If overconfidence is sufficiently common in the economy, i.e. if  $\kappa > \kappa_1(\lambda)$ , then the introduction of voluntary training programs is a Pareto improvement, as long as benefits  $\eta$  are sufficiently large and costs  $c$  are sufficiently low.*

**Proof.** For any  $\kappa$  fraction of overconfident agents, let  $\alpha^{LO}(\kappa)$  be the associated equilibrium contract as calculated in Proposition A.2. Suppose that in equilibrium all low-risk and overconfident agents join the program. For  $\eta$  large enough,  $\kappa' = (1 - \eta)\kappa < \kappa_2(\lambda)$ . By Proposition A.2, the low-risk equilibrium utility (and the overconfident agents' perceived utility)  $V(W, d, p_L, \alpha^{LO}(\kappa'))$  decreases in  $\kappa'$  when  $\kappa' \leq \kappa_2(\lambda)$ , and it is constant in  $\kappa'$  for  $\kappa' \geq \kappa_2(\lambda)$ . Hence, for  $c$  small enough,  $V(W - c, d, p_L, \alpha^{LO}(\kappa')) > V(W, d, p_L, \alpha^{LO}(\kappa))$ . This implies that (i) all low-risk and overconfident agents join the training program, hence verifying our equilibrium imputation, and (ii) in equilibrium low-risk agents benefit from the adoption of voluntary training programs. The check that also overconfident agents are better off is identical to the one in the proof of Result 4. The high-risk agents' equilibrium utility  $V(W, d; p_H, \alpha^H)$  is constant in  $\kappa$ . Because  $c > 0$ , they choose not to join training programs. ■

**Positive Results** This section formalizes and proves our positive results.

We begin with the comparison between overconfidence and risk aversion. We introduce a fraction  $\phi$  of high-risk, moderately risk-averse (type- $R$ ) agents in the model by Rothschild and Stiglitz (1996). When purchasing a contract  $\alpha$  their utility is  $\underline{V}(W, d; p_H, \alpha^H) = (1 - p_H)\underline{U}(W - \alpha_1) + p_H\underline{U}(W - d + \alpha_2)$ , with  $U'' < \underline{U}'' < 0$ . An immediate extension of the results by Rothschild and Stiglitz (1996) implies the following result.

**Result A.1** *Adding a small fraction of  $R$ -type individuals to the basic adverse-selection model reduces the insurance coverage.*

**Proof.** In equilibrium,  $\alpha^H = (p_H d, (1 - p_H)d)$ . When  $\phi = 0$ ,  $\alpha^L$  solves  $p_L \alpha_2^L = (1 - p_L) \alpha_1^L$  together with  $V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L)$ . When  $\phi > 0$ ,  $\alpha^R = \alpha^H$  and, because  $U'' < \underline{U}''$ , the incentive-compatibility constraint  $\underline{V}(W, d; p_H, \alpha^R) \geq \underline{V}(W, d; p_H, \alpha^L)$  is tighter than the constraint  $V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L)$ . Hence the coverage  $\alpha_1^L + \alpha_2^L$  is smaller when  $\phi > 0$ ,

and so is the average coverage  $(\alpha_1^R + \alpha_2^R)\phi + (\alpha_1^L + \alpha_2^L)\lambda + (\alpha_1^H + \alpha_2^H)(1 - \lambda - \phi)$ . ■

Turning to risk classification, we first note that the equilibrium contracts  $\alpha^{LO}(x)$  and  $\alpha^H(x)$  in any risk class  $x$  are determined by the parameters  $\kappa(x)$  and  $\lambda(x)$  according to the characterization in Propositions A.1 and A.2. Because each contract offered in each risk class must make zero profits within the risk class, the equilibrium in each risk class  $x$  is derived as if the risk class  $x$  was a single separate insurance market. Hence we obtain the following result.

**Result A.2** *For any  $x$  such that  $\kappa(x) > \kappa_2(\lambda(x))$ , the insurance price  $P^{LO}(x)$  increases in  $x$ , but  $P^H(x) > P^{LO}(x)$ , for every  $x$ .*

**Proof.** The equilibrium price of the high-risk agents' contract  $\alpha^H(x)$  is  $P^H(x) = p_H$ . When  $\kappa(x) > \kappa_2(\lambda(x))$ , the low-risk and overconfident individuals' contract  $\alpha^{LO}(x)$  satisfies the Zero-Profit condition  $p_{LO}(x)\alpha_2^{LO} = (1 - p_{LO}(x))\alpha_1^{LO}$ ; hence its price  $P^{LO}(x)$  is  $[\kappa(x)p_H + \lambda(x)p_L] / [\kappa(x) + \lambda(x)]$ . Evidently  $P^H(x) > P^{LO}(x)$ . ■

We conclude by stating formally and proving our result on the relationship between risk and coverage. As long as  $p_{OL}(x) - p_H$  is not too close to 1, the equilibrium characterization is analogous to the characterization in Propositions A.1 and A.2.<sup>29</sup>This leads to the following result.

**Result A.3** *Suppose that  $p_O > p_H$ , and  $p_{OL}(x) - p_H(x)$  is not close to 1, the relationship between insurance coverage and actual risk is negative (positive) if and only if  $p_{OL}(x) < (>)p_H$ . The relationship between insurance coverage and perceived risk is always positive.*

**Proof.** In equilibrium, high-risk individuals choose contract  $\alpha^H = (p_H d, (1 - p_H)d)$ , whereas low-risk and overconfident individuals choose contract  $\alpha^{LO}$  that satisfies the Zero-Profit condition  $p_{OL}(x)\alpha_2^{LO}(x) = (1 - p_{OL}(x))\alpha_1^{LO}(x)$ , and either the binding Incentive-Compatibility condition (A.2) when  $\kappa < \kappa_1^O(\lambda)$ , or the Tangency condition  $\frac{(1-p_L)U'(W-\alpha_1^{LO})}{p_L U'(W-d+\alpha_2^{LO})} = \frac{1-p_{LO}^O}{p_{LO}^O}$  when  $\kappa_1^O(\lambda) < \kappa < \kappa_2^O(\lambda)$ ; if  $\kappa > \kappa_2^O(\lambda)$ , then  $\alpha^L = 0$ .<sup>30</sup> Hence the low-risk and overconfident individuals' insurance coverage  $\alpha_1^{LO} + \alpha_2^{LO}$  is smaller than the high-risk individuals' coverage  $\alpha_1^H + \alpha_2^H = d$ . High-risk individuals' are riskier on average than low-risk and overconfident individuals if and only if  $p_{LO}^O < p_H$ . Because  $p_L < p_H$ , the relationship between insurance coverage and perceived risk is positive. ■

<sup>29</sup>When  $p_O > p_H$ , the condition that  $[(1 - p_L)/p_L] / [(1 - p_H)/p_H] > U'(W - d)/U'(W)$  is unduly strong and implies that  $\alpha^{LO}(x) = 0$  in equilibrium when  $p_{OL}(x) > p_H$ . We thus relax this condition and require only that

$$\frac{1 - p_O}{p_O} > \frac{(1 - p_L)U'(W - d)}{p_L U'(W)}.$$

When  $p_{OL}(x) - p_H$  is too large, it may that in equilibrium,  $\alpha_1^H + \alpha_2^H > d$ . High-risk individuals buy so much insurance that they are better off in the case of accident. We disregard this case as unrealistic.

<sup>30</sup>We omit the definitions of the thresholds functions  $\kappa_1^O$ ,  $\kappa_2^O$ , which only require obvious modifications in the definitions of the functions  $\kappa_1$ ,  $\kappa_2$ .

## B Appendix: Mechanism Design

Consider a large pool of individuals indexed in  $i$  who differ in actual risk  $p \in \{p_H, p_L\}$  and perceived risk  $\hat{p} \in \{p_H, p_L\}$ . Let the individual characteristics space be  $\Psi = \{p_H, p_L\} \times \{p_H, p_L\}$ . The characteristics distribution  $\rho$  is easily derived from the parameter  $\kappa$  and  $\lambda$ . An allocation is a state-contingent profile  $\alpha^* : \Psi \rightarrow \mathbb{R}_+^2$ , where  $A^* = \mathbb{R}_+^{2\Psi}$  is the set of allocations. An allocation is *second-best efficient* if there is no other feasible incentive-compatible allocation that Pareto improves it. An allocation  $\alpha^*$  is incentive compatible if

$$\hat{V}(\psi, \alpha^*(\psi)) \geq \hat{V}(\psi, \alpha^*(\psi')) \text{ for all } (\psi, \psi') \in \Psi^2, \quad (\text{B.1})$$

where the perceived expected utility of any type  $\psi = (p, \hat{p})$  with contract  $\alpha$  is  $\hat{V}(\psi, \alpha) = V(W, d; \hat{p}, \alpha)$ . The allocation  $\alpha^*$  is feasible if  $\sum_{\psi \in \Psi} \rho_\psi \pi^*(\psi, \alpha^*(\psi)) \leq 0$  where for any type  $\psi$ , the profit of a contract  $\alpha \in \mathbb{R}_+^2$  is  $\pi^*(\psi, \alpha) = \pi(p, \alpha)$ . Because of monotonicity of individuals' utilities, we can restrict attention without loss of generality to budget-balanced allocations  $\alpha^*$ :

$$\sum_{\psi \in \Psi} \rho_\psi \pi^*(\psi, \alpha^*(\psi)) = 0. \quad (\text{B.2})$$

When allowing for overconfidence, it is necessary to distinguish the maximization of actual or perceived utility. Let  $\gamma_\psi$  denote the weight of individuals with characteristics  $\psi$  in the welfare function. The “*political economy*” problem selects an allocation  $\alpha^*$  that maximize the perceived welfare  $\sum_{\psi \in \Psi} \gamma_\psi \hat{V}(\psi, \alpha^*(\psi))$ , subject to the constraints (B.1), (B.2) and to the *individual rationality* constraint  $\hat{V}(\psi, \alpha^*(\psi)) \geq V(\psi, \alpha^\psi)$  for all  $\psi \in \Psi$ ; where  $\alpha^\psi$  is the equilibrium contract of each type  $\psi$  (with a minor notational violation). The “*paternalistic*” problem maximizes the actual welfare  $\sum_{\psi \in \Psi} \gamma_\psi V^*(\psi, \alpha^*(\psi))$  subject to the constraints (B.1) and (B.2), and to the “*non-expropriation*” constraint  $V^*(\psi, \alpha^*(\psi)) \geq V(\psi, \alpha^\psi)$  for all  $\psi \in \Psi$ , where the actual utility of any type  $\psi = (p, \hat{p})$  with contract  $\alpha$  is  $V^*(\psi, \alpha) = V(W, d; p, \alpha)$ .

A mechanism designer would like to implement a second-best efficient allocation on the basis of the information revealed by the agents. Each individual *only knows* her perceived risk  $\hat{p}$ , and she (maybe mistakenly) believes that her actual risk  $p$  coincides with  $\hat{p}$ . She can only communicate her perceived ability  $\hat{p}$  to the mechanism-designer. Hence we restrict attention to allocations  $\alpha^*$  that are constant across the actual risk  $p$ . We let  $\bar{A} = \{\alpha^* \in A^* : \alpha^*(p_L, \hat{p}) = \alpha^*(p_H, \hat{p})\}$ , for any  $\hat{p} \in \{p_H, p_L\}$ . We can now formally restate and prove Result 3 in section 5.

**Result 3** *Suppose that  $\kappa > \kappa_1(\lambda)$ . The equilibrium outcome  $(\alpha^H, \alpha^{LO})$  maximizes low-risk individuals' utility among the second-best efficient individually-rational (and non-expropriatory) allocations  $\alpha^* \in \bar{A}$ .*

**Proof.** In order to Pareto improve the equilibrium  $(\alpha^H, \alpha^{LO})$ , any allocation  $\alpha^*$  must satisfy  $V(W, d; p_H, \alpha^*(H, H)) \geq V(W, d; p_H, \alpha^H)$ . In equilibrium  $\alpha^H \in \arg \max_\alpha V(W, d; p_H, \alpha)$  such that  $p_H \alpha_2 = (1 - p_H) \alpha_1$ . Hence, the candidate Pareto improvement allocation  $\alpha^*$  must satisfy  $p_H \alpha_2^*(H, H) \geq (1 - p_H) \alpha_1^*(H, H)$ . The contracts  $\alpha^*(L, L)$  and  $\alpha^*(H, L)$  coincide by construction. By the budget-balance condition (B.2) this constrains the terms of the contracts

$$p_{LO} \alpha_2^*(L, L) \leq (1 - p_{LO}) \alpha_1^*(L, L). \quad (\text{B.3})$$

But when  $\kappa > \kappa_1(\lambda)$ , in equilibrium,  $\alpha^{LO} = \arg \max_{\alpha} V(W, d; p_L, \alpha)$  such that  $\pi(p_L, \alpha) = 0$ . Hence, the allocation  $\alpha^*$  cannot be better than  $\alpha^{LO}$  for agents of type  $\psi = (L, L)$ , i.e.  $V(W, d; p_L, \alpha^*(L, L)) < V(W, d; p_L, \alpha^{LO})$ . This concludes that there is no individually-rational (or non-expropriatory) feasible allocation  $\alpha^*$  that improves upon the equilibrium outcome  $(\alpha^H, \alpha^{LO})$  for individuals of characteristics  $\psi = (L, L)$ , and that  $(\alpha^H, \alpha^{LO})$  is second-best efficient. ■

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